[redacted]

Ejercicio 1.

$\vec{r}(t) = (2t^2 - 3)\vec{i} + (4t + 4)\vec{j} + (t^3 + 2t^2)\vec{k}$ m.

posición $t=0 \rightarrow \vec{r}(t=0) = \underbrace{3\vec{i} + 4\vec{j}}$ m.

velocidad $t=0 \rightarrow \vec{v}(t) = \frac{d\vec{r}(t)}{dt} = (4t)\vec{i} + 4\vec{j} + (3t^2 + 4t)\vec{k}$ m/s

$\vec{v}(t=0) = \underbrace{0}$ m/s

velocidad $t=1 \rightarrow \vec{v}(t=1) = \underbrace{4\vec{i} + 4\vec{j} + 7\vec{k}}$ m/s

aceleración $t=2 \rightarrow \vec{a}(t) = \frac{d\vec{v}(t)}{dt} = 4\vec{i} + (6t + 4)\vec{k}$ m/s².

$\vec{a}(t=2) = \underbrace{4\vec{i} + 16\vec{k}}$ m/s²

componentes intrínsecas

- aceleración tangencial

X $\vec{a}_t = \frac{d|\vec{v}|}{dt} = \frac{d}{dt}\sqrt{(4t)^2 + 4^2 + (3t^2+4t)^2} = \frac{d}{dt}\sqrt{16t^2 + 16 + 9t^4 + 16t^2 + 24t^3} =$

$= \frac{d}{dt}\sqrt{9t^4 + 24t^3 + 32t^2 + 16}$ - - -→ ??

GENÉRICO

LEER !

ra t = 2s.

$\vec{a}_t = \frac{\vec{a}(t)\cdot\vec{v}(t)}{|\vec{v}(t)|} = \frac{(16t, 4, 18t^3 + 36t^2 + 16t)}{\sqrt{9t^4 + 24t^3 + 32t + 16}}$

$\hookrightarrow \vec{a}(t)\cdot\vec{v}(t) = 16t\,\vec{i} + 4\vec{j} + (\underbrace{18t^3 + 12t^2 + 24t^2 + 16t}_{18t^3 + 36t^2 + 16t})\vec{k}$

$\hookrightarrow |\vec{v}(t)| = \sqrt{9t^4 + 24t^3 + 32t^2 + 16}$

PARTICULAR
EN EL
PUNTO

$\hookrightarrow \vec{a}_t(t=2s) = \frac{d|\vec{v}|}{dt} = \frac{d}{dt}\sqrt{9\cdot(2)^4 + 24(2)^3 + 32(2)^2 + 16} = \frac{d}{dt}\sqrt{480}$ X

- aceleración normal:

$$a_n = \sqrt{|\vec{a}|^2 - a_t^2} = \sqrt{272 - 215'322} = \underline{7'528 \text{ m/s}^2}$$

$$\hookrightarrow |\vec{a}|^2 = \left(\sqrt{4^2 + (6t+4)^2}\right)^{\cancel{2}} = 16 + 36t^2 + 16 + 48t = 272$$

$$\hookrightarrow a_t^2 = \left(\frac{16t + 4 + 18t^3 + 36t^2 + 16t}{\sqrt{9t^4 + 24t^3 + 32t + 16}}\right)^2 = (1'46 + 0'18 + 14'6)^2 =$$

$$= 2'13 + 0'032 + 213'16 = 215'322$$

* Ejercicio 2.

$$\vec{r}(t) = (t + 4t^4)\vec{i} + (18 - 3t^2)\vec{j}$$

velocidad $t = 1s \rightarrow \vec{v}(t) = \frac{d\vec{r}(t)}{dt} = (1 + 16t^3)\vec{i} + (-6t)\vec{j}$

$$\vec{v}(t = 1s) = \underline{17\vec{i} - 6\vec{j} \text{ m/s}}$$

componentes cartesianas → x, y → OJO

aceleración $t = 1s \rightarrow \vec{a}(t) = \frac{d\vec{v}(t)}{dt} = 48t^2\vec{i} - 6\vec{j}$

$$\vec{a}(t = 1s) = \underline{48\vec{i} - 6\vec{j} \text{ m/s}^2}$$

componentes intrínsecas → a_t, a_n

$$a_t = \frac{\vec{a}(t) \cdot \vec{v}(t)}{|\vec{v}(t)|} = \frac{(\overbrace{48t^2 + 768t^5}^{816})\vec{i} + \overbrace{36t}^{36}\,\vec{j}}{\underbrace{\sqrt{1 + 256t^6 + 32t^3 + 36t^2}}_{18'02}} = \underline{45'28\vec{i} + 1'99\vec{j} \text{ m/s}^2}$$

$$a_n = \sqrt{|\vec{a}|^2 - a_t^2} = \underline{186'67 \text{ m/s}^2}$$

OJO! ESTO ESTÁ MAL XQ T = 1S.

$$\hookrightarrow |\vec{a}|^2 = \left(\sqrt{(48t^2)^2 + 6^2}\right)^{\cancel{2}} = 2304 \cdot (2)^4 + 6^2 = 36900$$

$$\hookrightarrow a_t^2 = 45'28^2 + 1'99^2 = 2054'24$$

de (x,y) es (-y,x),

(y,-x) → hay que

* EJERCICIO 3 → HECHO POR ÉL EN CLASE

3?

* EJERCICIO 4

$\vec{r}(t) = b \cdot \cos \Omega t \, \vec{i} + b \cdot \sin \Omega t \, \vec{j} + ct \, \vec{k}$ $\qquad$ $b, c, \Omega \rightarrow$ ctes. positivas.

$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = -\Omega \cdot b \cdot sen \Omega t \, \vec{i} + \Omega \cdot b \cdot \cos \Omega t \, \vec{j} + c \, \vec{k}$

$|\vec{v}(t)| = \sqrt{\Omega^2 b^2 \cdot sen^2 \Omega t \, \overset{1}{\cancel{\vec{i}^2}} + \Omega^2 b^2 \cdot \cos^2 \Omega t \, \overset{1}{\cancel{\vec{j}^2}} + c^2 \, \overset{1}{\cancel{\vec{k}^2}}}$

$= \sqrt{\Omega^2 b^2 \, (\underbrace{sen^2 \Omega t + \cos^2 \Omega t}_{1}) + c^2} = \sqrt{\Omega^2 b^2 + c^2}$

↑ sí es constante xq no depende de t.

$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = -\Omega^2 \cdot b \cos \Omega t \, \vec{i} - \Omega^2 b \, sen \Omega t \, \vec{j}$

$|\vec{a}(t)| = \sqrt{\Omega^4 b^2 \cos^2 \Omega t + \Omega^4 b^2 sen^2 \Omega t} = \sqrt{\Omega^4 b^2 (\underbrace{\cos^2 \Omega t + sen^2 \Omega t}_{1})} = \sqrt{\Omega^4 b^2}$

* EJERCICIO 5

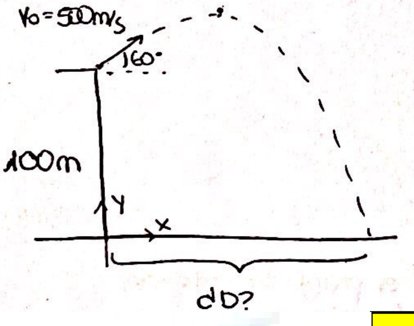
EC. PARAMÉTRICA $\begin{cases} x = R\,(\omega t + sen(\omega t)) \\ y = R\,(1 + \cos(\omega t)) \end{cases}$

↓

$\cos \omega t = \frac{y}{R} - 1 \Rightarrow \omega t = \arccos\left(\frac{y}{R} - 1\right)$

$x(y) = R\left(\arccos\left(\frac{y}{R} - 1\right) + sen\left(\arccos\left(\frac{y}{R} - 1\right)\right)\right)$ ← OPERAR CON ESTO

* EJERCICIO 6.



EJE X → MRU

$V_x = V_0 \cdot \cos\alpha = 250 m/s$

$X = X_0 + V_0 \cos\alpha t = 0 + 250 \cdot t$

EJE Y → MRUA

$V_y = V_0 sen\alpha - gt = 433 - 9'8t$

$y = y_0 + V_0 sen\alpha t - \frac{1}{2} gt^2 \Rightarrow$

$$\begin{cases} V_x = 250 m/s \\ V_y = 433 - 9'8t = -435'16 m/s \end{cases} \rightarrow |\vec{V}(t)| = \sqrt{V_x^2 + V_y^2} = \underbrace{501'86 m/s}$$

a) a 100 m de altura y bajando.

$$y = y_0 + v_0 \operatorname{sen}\alpha t - \frac{1}{2} g t^2$$

$$100 = 100 + 433t - \frac{1}{2} 9'8t^2 \rightarrow t(433 - 4'9t) = 0$$

$\hookrightarrow t \cancel{=} 0$

$\hookrightarrow 433 - 4'9t = 0$

$$t = \frac{433}{4'9} = 88'36 s$$

$$\left.\begin{array}{l} V_y = 433 - 9'8 \cdot 88'36 = -432'92 m/s \\ V_x = 250 m/s \end{array}\right\} |\vec{V}(t)| = 499'92 m/s$$

$$a_t = \frac{\overset{\overset{-9'8\vec{j}}{\uparrow}}{\vec{a}(t)} \cdot \overset{\overset{250\vec{i} - 432'92\vec{j}}{\uparrow}}{\vec{v}(t)}}{|\vec{V}(t)|} = \frac{(0, 4242'6)}{499'92} = \underbrace{8'48 m/s^2}$$

$$a_n = \sqrt{|\vec{a}|^2 - a_t^2} = \sqrt{96'04 - 71'91} = \sqrt{24'13} = \underbrace{4'91 m/s^2}$$

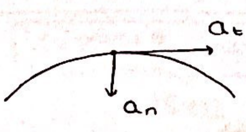
$\downarrow$

$$|\vec{a}| = \sqrt{0^2 + 9'8^2} = 9'8 \rightarrow |\vec{a}|^2 = 96'04$$

b) en el punto de altura máxima OJO

$a_t = \underbrace{0 m/s^2}$

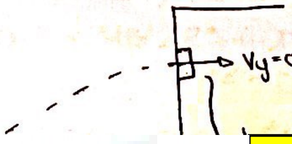
$a_n = \underbrace{-9'8 m/s^2}$



* Ejercicio 7.

¿Vo?

¿α?



↳ CONDICIÓN → Vy = 0 (que el proyectil entre de

EJE X → MRU

$$V_x = V_o \cdot \cos\alpha$$

$$D = 0 + V_o \cdot \cos\alpha t$$ ← sustituimos

EJE Y → MRUA

$$0 = V_o \operatorname{sen}\alpha - gt \Rightarrow t = + \frac{V_o \operatorname{sen}\alpha}{g}$$

$$h = 0 + V_o \operatorname{sen}\alpha t - \frac{1}{2}gt^2$$ sustituimos

$$D = 0 + V_o \cos\alpha t \longrightarrow D = 0 + V_o \cos\alpha \frac{V_o \operatorname{sen}\alpha}{g}$$

$$D = \frac{V_o^2 \cos\alpha \operatorname{sen}\alpha}{g} \Rightarrow \boxed{V_o^2 \operatorname{sen}\alpha = \frac{Dg}{\cos\alpha}}$$

$$h = 0 + V_o \operatorname{sen}\alpha \overset{\frac{V_o \operatorname{sen}\alpha}{g}}{t} - \frac{1}{2}g\overset{\frac{V_o^2 \operatorname{sen}^2\alpha}{g^2}}{t^2} \longrightarrow h = V_o^2 \operatorname{sen}^2\alpha \frac{1}{g} - \frac{1}{2}\frac{V_o^2 \operatorname{sen}^2\alpha}{g}$$

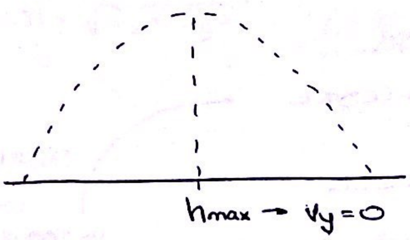
$$h = V_o^2 \operatorname{sen}^2\alpha \left(\underbrace{\frac{1}{g} - \frac{1}{2g}}_{\frac{1}{2g}}\right) \Rightarrow \boxed{V_o^2 \operatorname{sen}^2\alpha = 2gh}$$

$$\frac{\cancel{V_o^2} \operatorname{sen}^{\cancel{2}}\alpha}{\cancel{V_o^2} \cancel{\operatorname{sen}\alpha}} = \frac{2g h}{D \cancel{g}} \cdot \cos\alpha \Rightarrow \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{2h}{D} \cdot \frac{\cancel{\cos\alpha}}{\cancel{\cos\alpha}}$$

$$\tg\alpha = \frac{2h}{D}$$

★ EJERCICIO 8.

α para que $x = \text{máx}$



EJE X → MRU

$V_x = V_0 \cos\alpha$

$x = \cancel{x_0} + V_0 \cos\alpha\, t$

EJE Y → MRUA

$V_y = V_0 \operatorname{sen}\alpha - gt$

$y = y_0 + V_0 \operatorname{sen}\alpha t - \frac{1}{2} g t^2$

CONDICIÓN:

↳ $V_y = 0 = V_0 \operatorname{sen}\alpha - gt \Rightarrow t = \frac{V_0 \operatorname{sen}\alpha}{g}$

$x = \underbrace{x_0 + \frac{V_0^2}{g}}_{\text{constantes}} \frac{\operatorname{sen}\alpha \cos\alpha}{} \Rightarrow \underbrace{\frac{dx}{d\alpha} = 0}_{x = \text{max}} \Rightarrow$

derivada 1°, 2° sin derivar + 1° sin derivar · derivada 2°

$\cos\alpha \cos\alpha - \operatorname{sen}\alpha \operatorname{sen}\alpha = 0$

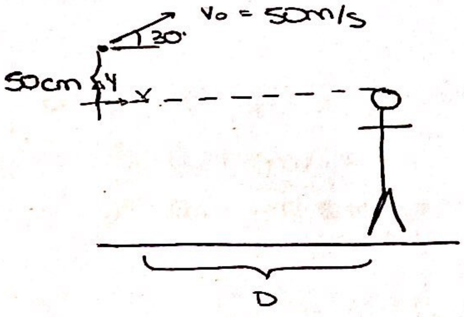
$\cos^2\alpha - \operatorname{sen}^2\alpha = 0$

$\cos^2\alpha = \operatorname{sen}^2\alpha$

↳ esto ocurre cuando

$\alpha = 45°$

★ EJERCICIO 9.



$a_y = 10 m/s^2$

$a_x = 2 m/s^2$

¿D?

¿hmax?

EJE X → MRUA

$V_x = V_0 \cdot \cos\alpha \ominus a_x t$ ↱ el viento va en contra

$x = x_0 + V_0 \cos\alpha t - \frac{1}{2} a_x t^2$

EJE Y → MRUA

$V_y = V_0 \operatorname{sen}\alpha - a_y t$

$y = y_0 + V_0 \operatorname{sen}\alpha t - \frac{1}{2} a_y t^2$

$0 = 0'5 + 50 \cdot \operatorname{sen}30 \cdot t - \frac{1}{2} \cdot 10 \cdot t^2$

$0 = 0'5 + 25t - 5t^2$

$t = \frac{-25 \pm \sqrt{625 + 10}}{} = \frac{-25 \pm 25'19}{} = \nearrow \boxed{5'019 s}$

$h_{max} \Rightarrow V_y = 0 \ m/s$

$$V_y = 0 = V_0 \operatorname{sen}\alpha - a_y t$$

$$0 = 50 \cdot \operatorname{sen}30 - 10 \cdot t \Rightarrow t = \frac{50\operatorname{sen}30}{10} = 2'5s.$$

$$y = h_{max} = 0'5 + \underbrace{50 \cdot \operatorname{sen}30 \cdot 2'5}_{62'5} - \underbrace{\frac{1}{2} 10 \cdot 2'5^2}_{31'25} = 31'75m$$

→ Cuando $y = 25m$ e $y_0 = 1'8m$.

$$25 = 1'8 + 50 \cdot \operatorname{sen}30 \cdot t - 5 \cdot t^2$$

$$-5t^2 + 25t - 23'2 = 0$$

$$t = \frac{-25 \pm \sqrt{625 - 464}}{-10} = \frac{-25 \pm 12'7}{-10} = \begin{cases} 3'77seg \\ 1'23\,seg. \end{cases}$$

← cogemos este xq te dice cuando baja por lo tanto mayor tiempo.

$$\left.\begin{array}{l} V_x = 50 \cdot \cos30 - 2 \cdot 3'77 = 35'76 \ m/s \\ V_y = 50 \cdot \operatorname{sen}30 - 10 \cdot 3'77 = -12'7 \ m/s \end{array}\right\} |\vec{V}| = 37'94 \ m/s$$

$$\vec{a}_t = \frac{\overset{(2,10)}{\vec{a}} \cdot \overset{(35'76,-12'7)}{\vec{V}}}{|\vec{V}|} = \frac{(71'52, -127)}{37'94} = (1'88, -3'34) \rightarrow a_t = 3'83 \ m/s^2$$

$$a_n = \sqrt{|\vec{a}|^2 - a_t^2} = 9'45 \ m/s^2$$

$$\downarrow$$

$$|\vec{a}| = \sqrt{2^2 + 10^2} = 10'19$$

$$|\vec{a}|^2 = 104$$

* EJERCICIO 10. HECHO AL FINAL DEL TODO !!

37. EJE X → MRU.

$$V_x = V_0 \cdot \cos\alpha$$

$$x = x_0 + V_0 \cos\alpha t \rightarrow x = V_0 \cos\alpha t \Rightarrow t = \frac{x}{}$$

$$V_{em} = \frac{mg}{b} \quad V_{em} = \sqrt{\frac{mg}{C}}$$

$b \rightarrow \propto D \qquad C \rightarrow \propto D^2$

* EJERCICIO 15

Froz cuadrática
"cualquier forma" $= \underbrace{\frac{1}{2}\rho\cdot \overset{D^2}{\overset{\uparrow}{A}}\cdot C_d}_{C}\cdot v^2$

peso $= m\cdot g = \rho\cdot V\cdot g$

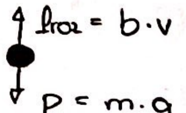
froz = peso

$$\frac{1}{2}C_D\cdot v^2\cdot\rho\cdot \overset{\cancel{\pi}\cancel{r^2}}{\overset{\uparrow}{D^2}} = \rho\cdot \overset{\frac{4}{3}\cancel{\pi} r^{\cancel{3}}}{\overset{\uparrow}{V}}\cdot g$$

$$\frac{1}{2}C_D\cdot v^2\cdot\rho = \frac{4}{3}\rho\cdot r\cdot g$$

$$v = \sqrt{\frac{4\cancel{\rho} r g\cdot 2}{3\cancel{\rho}\cdot \underset{0'5}{\underset{\shortparallel}{C_D}}}} = 6'26\cdot 10^{-3}\ m/s$$

* EJERCICIO 16.



v > vem

→ mg > Froz ⟶ se acelera ⇒ v↑ , Froz ↑

→ mg = Froz ⟶ v = cte

→ mg < Froz ⟶ se frena ⇒ v↓ , Froz ↓ .

nuestro caso porque v > Vem
por lo que v va disminuyendo
hasta Vem.

segun pasa el tiempo, la velocidad va disminuyendo hasta alcanzar vem.

$$m\cdot \overset{a}{\frac{dv}{dt}} = mg - b\cdot v = -b\left[v - \frac{mg}{v}\right] = -b\left[v - V_{em}\right]$$

$$v - V_{em} = u$$

$$\frac{dv}{dt} = \frac{du}{dt}$$

$$m\cdot\frac{du}{dt} = -b\cdot u \Rightarrow \int\frac{du}{dt} = -\frac{b}{m}\cdot u = \int -Ku$$

$$\underset{\underset{v - V_{em}}{\downarrow}}{u} = A\cdot e^{-Kt} = A\cdot e^{-t/z}$$

$$v - V_{em} = A\cdot e^{-t/z}$$

$$v = V_{em} + A\cdot e^{-t/z}$$

cuando $\underset{V_{y_0}}{\underbrace{v(0)}} = V_{em} + A \Rightarrow A = V_{y_0} - V_{em}$

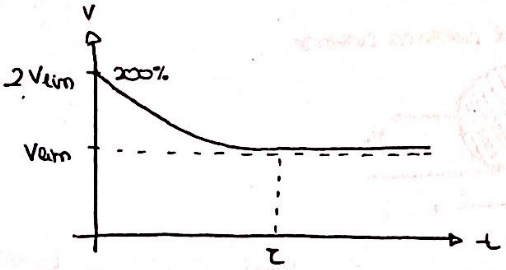
$$\ldots \quad v = V_{em}\,(e^{-t/\tau} + 1)$$

??

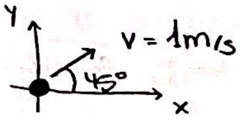
- $t = \tau \Rightarrow 136\%$
- $t = 2\tau \Rightarrow 113\%$
- $t = 3\tau \Rightarrow 104\%$
- $t = 4\tau \Rightarrow 101\%$
- $t = 5\tau \Rightarrow 100'6\%$

↓

se va acercando a Vem



* EJERCICIO 18.



$d = 0'2mm$

$\rho = 16 g/cm^3$ = $16000\ kg/m^3$

alcance ↑

$$R = R_{vac}\left(1 - \frac{4V_{y_0}}{3V_{em}}\right) = 9'73cm$$

$$V_{y_0} = V_0 \cdot \operatorname{sen}\alpha = 1 \cdot \operatorname{sen}45° = 0'7$$

$$V_{em} = \frac{m \cdot g}{b} = \frac{V \cdot \rho \cdot g}{b} = \frac{\frac{4}{3}\pi r^3\, \rho \cdot g}{\underset{\downarrow\ d \cdot \beta = 3'2 \cdot 10^{-8}}{b}} = 20'52 m/s$$

$$R_{vac} = \frac{2V_{x_0}V_{y_0}}{g} = 10'2cm$$

En el caso de la pelota de aluminio → R = 7'44cm

* EJERCICIO 19.

$$V_{em} = \sqrt{\frac{mg}{C}} = \sqrt{\frac{mg}{\gamma \cdot D^2}} = \sqrt{\frac{0'15 \cdot 9'8}{0'25 \cdot 0'07^2}} = 34'64 m/s \leftarrow \text{PELOTA}$$

$$V_{em} = \sqrt{\frac{mg}{C}} = \sqrt{\frac{0'9 \cdot 10^{-3} \cdot 9'8}{2'25 \cdot 10^{-6}}} = 22'19 m/s \leftarrow \text{BOLA ACERO}$$

$$\hookrightarrow \rho = \frac{m}{V} \Rightarrow m = \rho \cdot V = \underset{g/cm^3}{\rho} \cdot \underset{cm^3}{\frac{4}{3}\pi r^3} = 0'90 g = 0'904 \cdot 10^{-3} kg$$

* EJERCICIO 20.

FÓRMULA

Sabemos que $v(t) = V_0$