Ejercicios

9.1. Escribiéndolas como series telescópicas, estudiar las siguientes series:

a) $\sum_{n=1}^{\infty} \frac{1}{2^n} \cdot \frac{n+2}{n(n+1)}$. (Descomponer $\frac{n+2}{n(n+1)}$ en fracciones simples.)

b) $\sum_{n=1}^{\infty} 3^n \operatorname{sen}^3 \frac{a}{3^n}$. (Obsérvese que $\operatorname{sen} x = 3 \operatorname{sen} \frac{x}{3} - 4 \operatorname{sen}^3 \frac{x}{3}$.)

c) $\sum_{n=1}^{\infty} 2^{n-1} \tan^2 \frac{a}{2^n} \tan \frac{a}{2^{n-1}}$ (Utilizar que $\tan x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}}$.)

d) $\sum_{n=1}^{\infty} \operatorname{sen} \frac{1}{2^n} \cos \frac{3}{2^n}$. (Tener en cuenta que $\cos x \operatorname{sen} y = \frac{1}{2}(\operatorname{sen}(x+y) - \operatorname{sen}(x-y))$.)

e) $\sum_{n=1}^{\infty} \frac{1}{4^n \cos^2(x/2^n)}$, $0 < x < \pi/2$. (Usar que $\frac{1}{4\cos^2 a} = \frac{1}{\operatorname{sen}^2 2a} - \frac{1}{4 \operatorname{sen}^2 a}$.)

9.2. Estudiar el carácter de las siguientes series:

1) $\sum_{n=1}^{\infty} \frac{\operatorname{sen}^4 n}{n^2}$,

2) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} - 2/3}$,

3) $\sum_{n=1}^{\infty} \frac{1+n^2}{n!}$,

4) $\sum_{n=1}^{\infty} \cos^n (a + \frac{b}{n})$, $0 < a < \pi/2$,

5) $\sum_{n=1}^{\infty} \frac{n^2+1}{na^n}$, $a \neq 0$,

6) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$,

7) $\sum_{n=1}^{\infty} \frac{3^n}{n^2+1}$,

8) $\sum_{n=1}^{\infty} (\frac{n+1}{n})^{-n^3}$,

9) $\sum_{n=1}^{\infty} \frac{1}{\log n}$,

10) $\sum_{n=1}^{\infty} \frac{1}{na+b}$, $(a,b) \neq (0,0)$,

11) $\sum_{n=1}^{\infty} \frac{\operatorname{sen} nx}{n^2}$,

12) $\sum_{n=1}^{\infty} \frac{1}{n - 3/2}$,

13) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$,

14) $\sum_{n=1}^{\infty} \frac{1 + \operatorname{sen}^2 nx}{n^2}$,

15) $\sum_{n=1}^{\infty} \frac{1}{n} \operatorname{sen} \frac{1}{n}$,

16) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n}$,

17) $\sum_{n=1}^{\infty} \frac{n(n+1)}{n^2+2n}$,

18) $\sum_{n=1}^{\infty} (\frac{1}{n})^{n+1/n}$,

19) $\sum_{n=1}^{\infty} \frac{\log(n+1) - 1}{(1+n)^2}$,

20) $\sum_{n=1}^{\infty} \frac{1}{3 - \cos(1/n)}$,

21) $\sum_{n=1}^{\infty} (\frac{x}{n})^n n!$,

22) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(1+\frac{1}{2}+\cdots+\frac{1}{n})}$, 27) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(\log n)^p}$, 32) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n+1)}{n!}$,

23) $\displaystyle\sum_{n=1}^{\infty} \frac{1+\frac{1}{2}+\cdots+\frac{1}{n}}{n^3\log n}$, 28) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$, 33) $\displaystyle\sum_{n=1}^{\infty} \frac{(n^2+1)x^n}{(n+1)!}$,

24) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(\log n)^{2n}}$, 29) $\displaystyle\sum_{n=1}^{\infty} \frac{\log n}{n^p}$, 34) $\displaystyle\sum_{n=1}^{\infty} e^{1/n^2} - e^{1/(n^2+1)}$,

25) $\displaystyle\sum_{n=1}^{\infty} \log\frac{n+1}{n}$, 30) $\displaystyle\sum_{n=1}^{\infty} \log\big(1+\frac{x}{n}\big)$, 35) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}\frac{n}{n^2+1}$,

26) $\displaystyle\sum_{n=1}^{\infty} e^{-\sqrt{n^2+1}}$, 31) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\frac{1}{2}+\cdots+\frac{1}{n}}$, 36) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}x^{2n}$.

9.3. Hallar la suma, si converge, de las siguientes series:

a) $\displaystyle\sum_{n=2}^{\infty} \frac{4n-1}{(n+2)(n-1)^2}$,

b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$,

c) $\displaystyle\sum_{n=2}^{\infty} \frac{2n+3}{n(n-1)(n+2)}$,

d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2-1}$,

e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n^2+16n+7}$,

f) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(n+1)^2-4}$,

g) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2+7n+6}{n(n+1)(n+2)(n+3)}$,

h) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n-1+\sqrt{3})(n-2+\sqrt{3})(n+\sqrt{3})}$

i) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+3n+1}{n^2(n+1)^2}$,

j) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2(n+1)^2}{n!}$,

k) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n(n-3)}{n!}$,

l) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3-n+1}{n!3^n}$,

m) $\displaystyle\sum_{n=3}^{\infty} \frac{3n^2+8n+6}{(n+2)!}$,

n) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n!(n+2)}$,

ñ) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3-1}{n!}$,

o) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{(n+1)!}$,

p) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2+5n+7}{(n+2)!}$,

q) $\displaystyle\sum_{n=1}^{\infty} (-1)^n\frac{2n+1}{n(n+1)}$,

r) $\displaystyle\sum_{n=1}^{\infty} \frac{n(n+1)}{2^n}$,

s) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^n}$,

t) $\displaystyle\sum_{n=1}^{\infty} (n+1)x^n$,

u) $\displaystyle\sum_{n=1}^{\infty} (-1)^n\frac{n^2-n}{3^n}$,

v) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}+(n+1)\sqrt{n}}$,

w) $\displaystyle\sum_{n=1}^{\infty} \Big(\log\Big(\frac{n}{n+1}\Big)^n - \frac{1}{2n}+1\Big)$.

9.4. Hallar la suma de las siguientes series:

a) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots,$

b) $\frac{1}{4} - \frac{1}{3} + \frac{1}{8} - \frac{1}{9} + \frac{1}{12} - \frac{1}{15} + \frac{1}{16} - \frac{1}{21} + \frac{1}{20} - \cdots,$

c) $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} - \frac{1}{2} + \frac{1}{9} + \frac{1}{11} + \frac{1}{13} + \frac{1}{15} - \frac{1}{4} + \cdots,$

d) $1 + \frac{1}{3} - \frac{1}{2} - \frac{1}{4} - \frac{1}{6} + \frac{1}{5} + \frac{1}{7} - \frac{1}{8} - \frac{1}{10} - \frac{1}{12} + \cdots,$

e) $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \cdots$