# Topic 1: <u>Review of Stochastic Processes</u>

## Academic Year 2013 - 2014

**P1.-** Let the stochastic process be defined by the set of all possible realizations $x[n]$, $-5 \leq n \leq 5$, generated by 11 independent releases of a 6-sided die and values $x \epsilon \{1,2,3,4,5,6\}$. Obtain:

 a) The PDF of $x[n]$ in the time instant $n_1$. What is the probability of observing a realization such that $x[0] = 3$?

 b) The joint distribution function in the time instants $n_1$ and $n_2$. What is the probability of observing a realization such that $x[-1] = 3$ y $x[1] = 3$?

 c) The probability of obtaining the value 1 for all $-5 \leq n \leq 5$.

 d) The probability of obtaining realizations $x[n] > 3$ for all $-5 \leq n \leq 5$.

**P2.-** Let the stochastic process be defined by the set of all possible realizations $x[n]$, generated by independent measurements of a process which values are distributed uniformly as $X \sim U(-1,1)$. Obtain:

 a) The PDF of the magnitude $x[n]$ at time instant $n_1$.

 b) The joint PDF of the magnitude $x[n]$ at the time instants $n_1$ and $n_2$, where $n_1 \neq n_2$.

 c) The probability that given a time instant $n_1$, $0 < x[n_1] < 0.5$.

 d) The probability that given two different time instants $n_1$ and $n_2$, $0 < x[n_1] < 0.5$ and $-0.5 < x[n_2] < 0$.

**P3.-** The PDF of the stochastic, stationary and ergodic process $x(t)$, $f(x_1; t_1)$, is a uniform function $X \sim U(1, 2)$. Obtain:

 a) The fraction of time in which $x(t) > 1.5$.

 b) The fraction of time in which $x(t) < 1.75$.

 c) The mean value of $x(t)$.

 d) The mean power of $x(t)$.

**P4.-** What is the autocorrelation $R_{yy}(\tau)$ of an i.i.d. process ("independent and identically distributed" ➔ "independent and stationary") $y[n]$ whose samples are distributed according to $Y \sim N(m_y, \sigma_y^2)$? What is the physical meaning of $R_{yy}(0)$?

**P5.-** Suppose now that the samples from the process $z[n]$ are distributed according to a random variable $Z$. If $Z$ is a function of the random variable $Y \sim N(m_y, \sigma_y^2)$, namely $Z = aY + b$, where $a$ and $b$ are constants, what is the autocorrelation $R_{zz}(\tau)$? What is its physical meaning?

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**P7.-**  Obtain the power or energy spectral density (as appropiate) of the signal defined as:

$$x(t) = \begin{cases} 1, & |t| < T \\ 0, & resto \end{cases}$$

**P8.-**  Obtain the mean value, the energy, the power, the autocorrelation function and the power spectral density of $x$(t) defined as:

$$x(t) = cos(2\pi f t + \varphi)$$

**P9.-**  Let $n(t)$ be an uncorrelated Gaussian noise, whose magnitude is distributed with mean 0 and variance $\sigma^2$. Obtain the PDF (as a function of time, i.e. at each instant $t$), the mean value, the power, the autocorrelation and power spectral density of the signal:

$$s(t) = n(t) + cos(2\pi f t)$$

**P10.-**  Let $n(t)$ be a process whose power spectral density takes a constant value between $\pm f_c$-$B$/2 and $\pm f_c$+$B$/2, with $B \ll f_c$ and is zero for any frequency outside these two intervals. The total power of the noise is $P_n$. Consider this signal uncorrelated with any other. Obtain the spectrum of the signal:

$$s(t) = n(t) + cos(2\pi f_c t).$$