Topic 3: <u>Signal Representation</u>

<u>in Communication Systems</u>

Academic Year 2013 - 2014

P1.-   How can you recover a band-pass signal from its low-pass equivalent? Prove it mathematically.
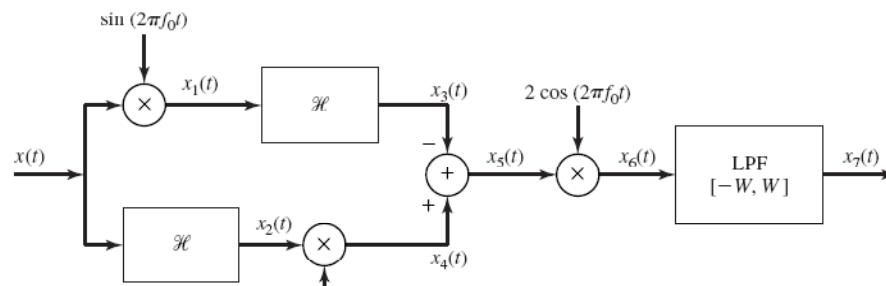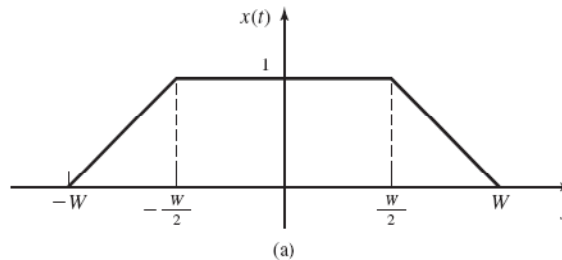
P2.-   Use a phasors diagram to obtain the expressions for the in-phase and quadrature components ($x_i(t)$ and $x_q(t)$); and the envelope and instantaneous phase ($A(t)$ and $\phi(t)$) of the band-pass signal:

$$x(t) = x_1(t)\cdot\cos(w_0 t) + x_2(t)\cdot\cos(w_0 t + \alpha)$$

P3.-   Repeat the previous exercise for the signal:

$$x(t) = x_1(t)\cdot\cos((w_0 - w_1)t) + x_2(t)\cdot\cos((w_0 + w_1)t)$$

P4.-   A low-pass signal $x(t)$ whose Fourier transform is shown in Figure 4(a) passes through the system shown in Figure 4(b) (where H blocks represent Hilbert transforms). Assuming that $W << f_0$, represent in frequency the signals obtained at each point of the system.

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P6.- Suppose a low-pass signal $m(t) = \text{sinc}^2(t)$ and a band-pass signal $x(t) = m(t)\cos(2\pi f_0 t) - \hat{m}(t)\sin(2\pi f_0 t)$:

 a) Obtain the analytical signal associated to $x(t)$ and low-pass equivalent of $x(t)$.
 b) Obtain and represent the Fourier transform of the signal $x(t)$. (Remember that $TF\{m(t)\} = \Lambda(f)$.)
 c) Repeat the two previous sections with $x(t) = m(t)\cos(2\pi f_0 t) + \hat{m}(t)\sin(2\pi f_0 t)$.

P7.- Assuming a center frequency $f_c = 1200Hz$, obtain the low-pass equivalent and the in-phase and quadrature components of the signal:

$$X(f) = \begin{cases} 1 & 900Hz \leq |f| < 1300Hz \\ 0 & resto \end{cases}$$

P8.- Suppose the next signals:

$$g(t), \text{ with } TF\{g(t)\} = \Pi(f)$$

$$x(t) = 2 \cdot g(t) \cdot \cos\left(2\pi \cdot f_0 \cdot t + \frac{\pi}{3}\right) \qquad\qquad y(t) = 2 \cdot g(t) \cdot \sin\left(2\pi \cdot f_0 \cdot t + \frac{\pi}{2}\right)$$

$$p(t) = x(t) + y(t)$$

 a) Obtain the analytical signal associated to $x(t)$ and its low-pass equivalent.
 b) Obtain the signal $\hat{y}(t)$ (i.e., the Hilbert Transform of $y(t)$). Obtain and represent the $TF\{\hat{y}(t)\}$.
 c) Use a phasors diagram to obtain the expressions for the in-phase and quadrature components ($p_i(t)$ and $p_q(t)$); and the envelope and instantaneous phase ($A(t)$ and $\phi(t)$) of the signal $p(t)$.

Note: $\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
   $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

P9.- Assuming a center frequency $f_c = 1200Hz$ and the signal:

$$X(f) = \begin{cases} 1 & 1100Hz \leq |f| < 1200Hz \\ 0.5 & 1200Hz \leq |f| < 1350Hz \\ 0 & resto \end{cases}$$

 a) Obtain the low-pass equivalent.
 b) Obtain the in-phase and quadrature components ($x_i(t)$ and $x_q(t)$).
 c) Obtain the signal $\hat{x}_i(t)$ (i.e., the Hilbert transform of the signal $x_i(t)$).

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Departamento de Teoría de la Señal y Comunicaciones.
Fundamental Principles of Telecommunication Systems.

Universidad Rey Juan Carlos.
Problems Topic 3.

P10.- A signal $x(t)$ with Fourier transform $X(f) = \prod\left(\frac{f}{1000}\right)$ passes through the system shown in Figure 10 (where H blocks represent Hilbert transforms). Assuming that $W = $ 1KHz and $f_0 = 10$KHz:
  a) Obtain (in time and frequency; only up to the signal $x_5(t)$) and represent in frequency the signals obtained at each point of the system.
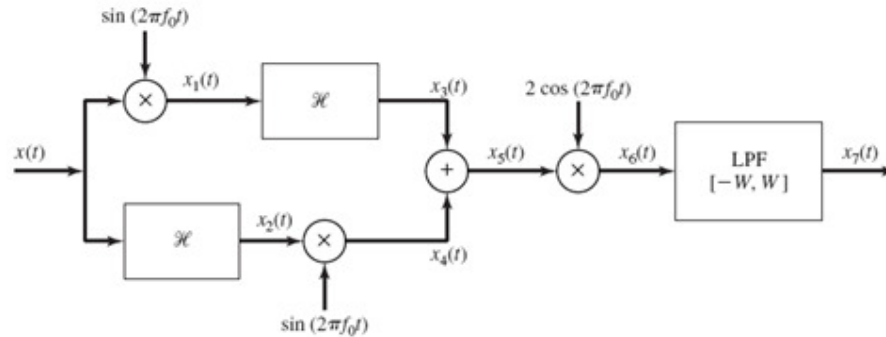 b) Obtain the analytical signal associated to $x_5(t)$ and its low-pass equivalent.

Figure 10.

Note: You can leave the solutions of this problem in terms of $x(t)$ and TH$\{x(t)\}$.

P11.- Suppose the next signals:

$$g(t), \text{ with } TF\{g(t)\} = \Pi(f)$$

$$x(t) = 2 \cdot g(t) \cdot \cos\left(2\pi \cdot f_0 \cdot t + \frac{\pi}{3}\right) \qquad y(t) = 2 \cdot g(t) \cdot \sin\left(2\pi \cdot f_0 \cdot t + \frac{\pi}{2}\right)$$

$$z(t) = -2 \cdot g(t) \cdot \cos\left(2\pi \cdot f_0 \cdot t - \frac{\pi}{3}\right) \qquad p(t) = x(t) + y(t) + z(t)$$

  a) Use a phasors diagram to obtain the expressions for the in-phase and quadrature components ($p_i(t)$ and $p_q(t)$); and the envelope and instantaneous phase ($A(t)$ and $\phi(t)$) of the signal $p(t)$.
  b) Obtain the analytical signal associated to $x(t)$ and its low-pass equivalent.
  c) Obtain and represent the $TF\{\hat{g}(t)\}$ (i.e., the Fourier transform of the signal $g(t)$).
  d) Obtain the signal $\hat{y}(t)$ (i.e., the Hilbert transform of the signal $y(t)$). Obtain and represent the $TF\{\hat{y}(t)\}$.

Note:

$$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$