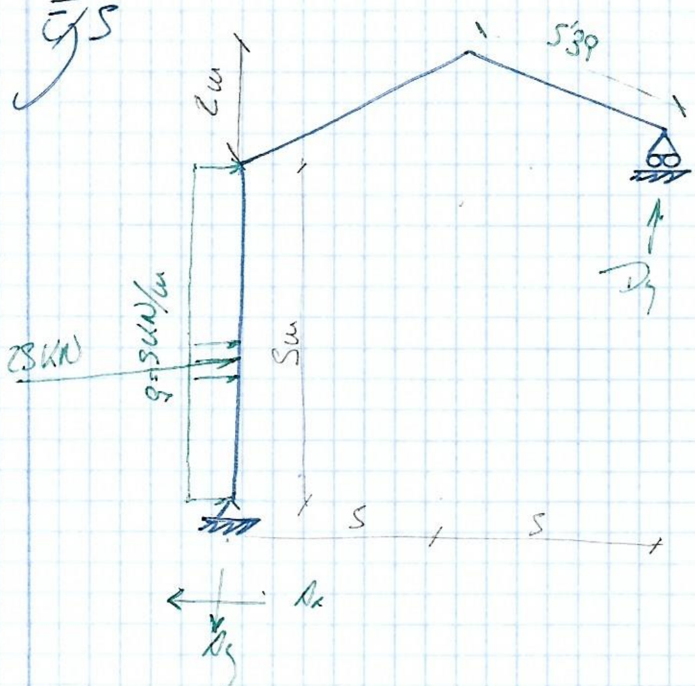
$\sin 21,8 = \frac{L}{7}$ $T = 5,39$

Ej 5

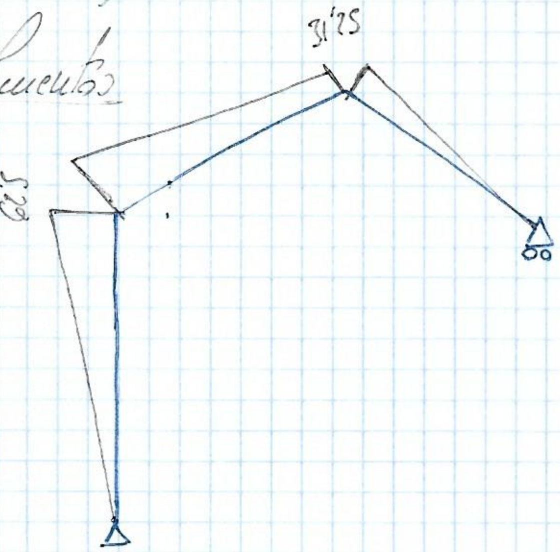


$\Sigma F_x = 0 \quad -A_x + 25 = 0$
$\Sigma F_y = 0 \quad -A_y + D_y = 0$
$\Sigma M_A = 0 \quad -25 \cdot 2'5 + D_y \cdot 10 = 0$

$A_x = +25\ kN$
$A_y = +6,25\ kN$

$D_y = 6,25\ kN$

Momentos



M_A: $M_A = 0$

M_B: $M_B - 25 \cdot 5 + 25 \cdot 2'5 = 0$

$M_B = 62,5$

M_C: $M_C + 6,25 \cdot 5 + 25 \cdot 4'5 - 25 \cdot 2 = \ldots$

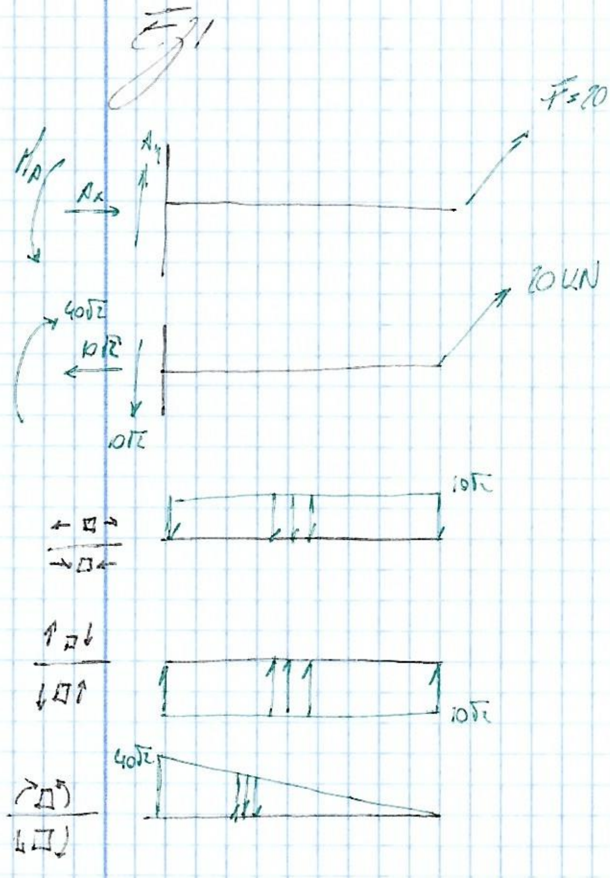
$M_C = 31,25$

M_D: $M_D = 0$

$$u_{Dx} = \left(\frac{5 \cdot 62'5}{2} \cdot 10\right) + \left(\frac{5'39 \cdot 62'5}{2} \cdot 1'66\right) + \left(31'25 \cdot 31'25 \cdot 2'50\right) + \left(\frac{5 \cdot 62'5}{2} \cdot 3'34\right) =$$

$$= \frac{4805}{EI}$$

Ej 1



(x) $\Sigma F_x = 0$; $A_x + F_x = 0$; $A_x = -F_x = -10\sqrt{2}\,kN$

(y) $\Sigma F_y = 0$; $A_y + F_y = 0$; $A_y = -F_y = -10\sqrt{2}\,kN$

(M_A) $\Sigma M_A = 0$; $M_A + F_y \cdot 4 = 0$; $M_A = -10\sqrt{2}\cdot 4 = -40\sqrt{2}\,kN$

Desplazamiento Horizontal (u)

$$du = \frac{N}{EA}\cdot dx \; ; \; u = \int_0^L \frac{N}{EA}\cdot dx \rightarrow u = \frac{F\sqrt{2}}{2EA}\cdot L$$

$$u = \frac{10\sqrt{2}}{20\cdot 10^6 \cdot d^2}\cdot 4 = 5'02\cdot 10^{-5}\,m$$

Desplazamiento vertical (V)

$$dV = \frac{T}{GA_c}\cdot dx \; ; \; V = \int_0^L \frac{T}{GA_c}\cdot dx \rightarrow V = \frac{F\sqrt{2}}{2GA_c}\cdot L$$

$$V = \frac{10\sqrt{2}}{8'5\cdot 10^6 \cdot 0'653}\cdot 4 = 1'3\cdot 10^{-4}$$

Giro θ a partir del momento M

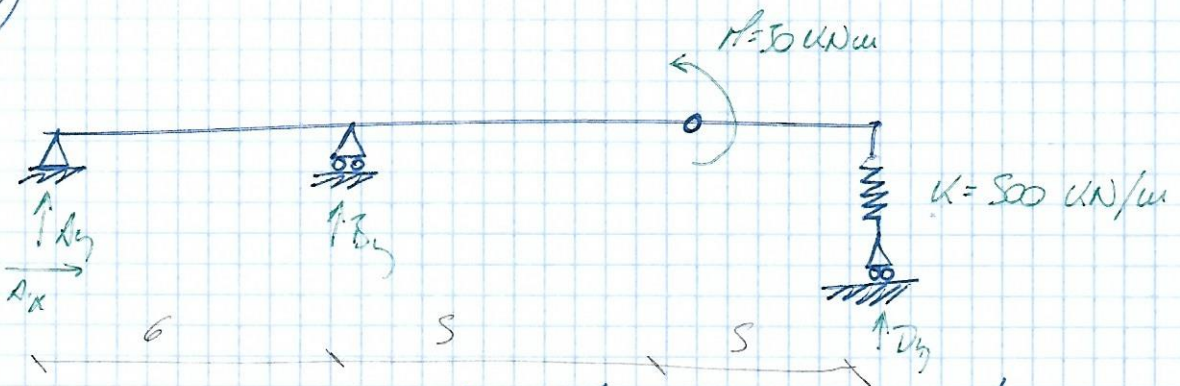
$$\theta = \frac{10\sqrt{2}\cdot 4}{2\cdot 20\cdot 10^6\cdot 1'7\cdot 10^{-4}} = 0'042\,rad$$

Desplazamiento vertical V_M debido al momento M

$$dV_M = d\theta\cdot(x_B - x) \; ; \; V_M = \int_0^L \frac{M}{E\cdot I}\cdot(L-x)\cdot dx \rightarrow V_M = M_A\cdot d \text{ (centro)}$$

$$V_M = \sqrt{2}\,F\cdot L\cdot\frac{L}{2}\cdot\frac{2}{3}\cdot L = 40\sqrt{2}\cdot\frac{4}{2}\cdot\frac{2}{3}\cdot 4 = 8'02\cdot 10^{2}\,m$$

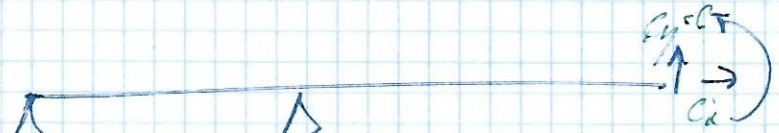
Ej 2



a) Diagramas de momentos flectores y esfuerzos cortantes

b) Giro en los apoyos A_y y B

c) Movimiento vertical de la articulación C

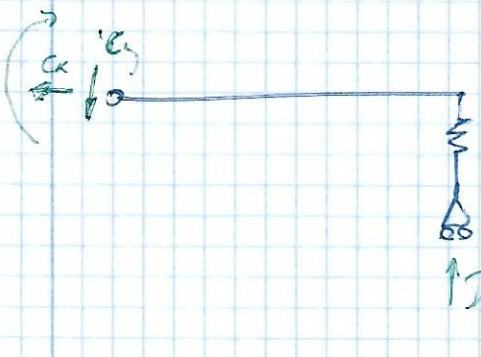


$\Sigma F_x = 0 \quad A_x + C_x = 0 \quad A_x = 0$

$\Sigma F_y = 0 \quad -B_y + A_y + C_y = 0 \quad A_y =$

$\Sigma M_A = 0 \quad -B_y \cdot C + 6 \cdot 11 + 30 = 0$

$B_y = 16$

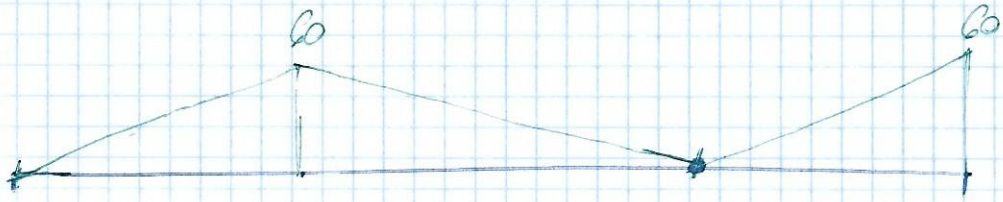


$\Sigma M_C = 0 \quad 30 + D_y \cdot 5 = 0 \rightarrow D_y = 6\,kN$

$\Sigma F_x = 0 \quad C_x = 0$

$\Sigma F_y = 0 \quad 6 - C_y = 0 \quad \boxed{C_y = C}$

Momentos



(B) $M_B + 30 + 10 \cdot 6 = 0$

$M_B = C\Theta$

Giro en A y en B (Navier)

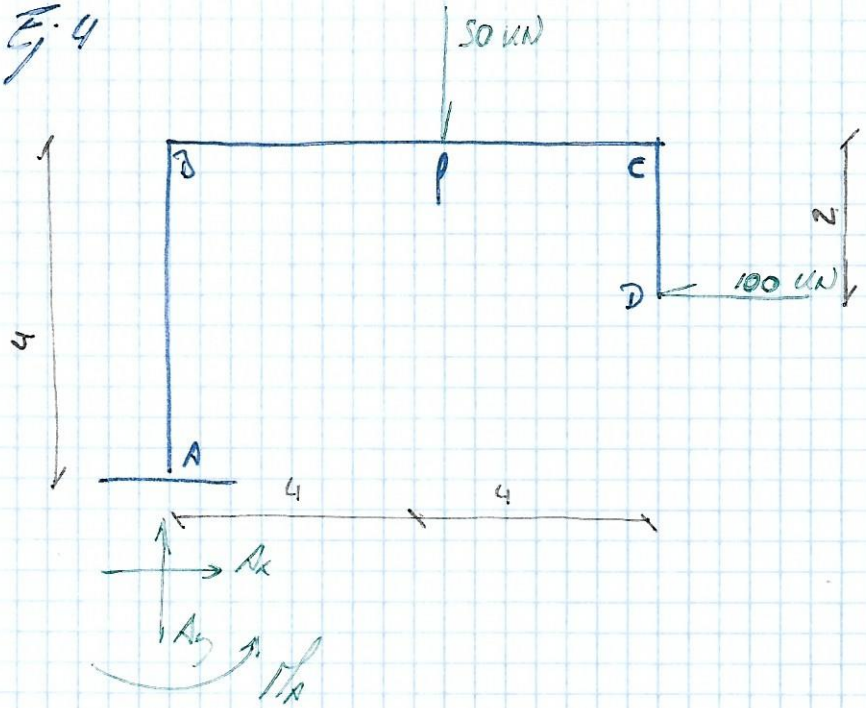
1º) $0 = \theta_A \cdot 6 + \frac{1}{EI} \cdot \frac{6 \cdot 30}{2} \cdot 2 - \frac{10^{-5} \cdot 40^{\circ}C}{0'8} \cdot \frac{6^2}{2} \rightarrow \theta_A = 1'5 \cdot 10^{-3}$ rad

2º) $\theta_B = \frac{1'5 \cdot 10^{-3}}{\theta_A} - \frac{1}{EI} \cdot \frac{30 \cdot 6}{2} - \frac{10^{-5} \cdot 40}{0'8} \cdot 6 = 1'2 \cdot 10^{-3}$ rad

Desplazamiento vertical → con referencia en A

$y_C = -1'6 \cdot 10^{-3} \cdot 5 + \frac{1}{EI} \cdot \frac{30 \cdot 5}{2} \cdot \frac{2}{3} \cdot 5 - \frac{10^{-5} \cdot 40}{0'8} \cdot \frac{5^2}{2} = -0'015$ m

Ej. 4



a) Dibujar y acotar los leyes de esfuerzos flectores indicando los puntos Singu

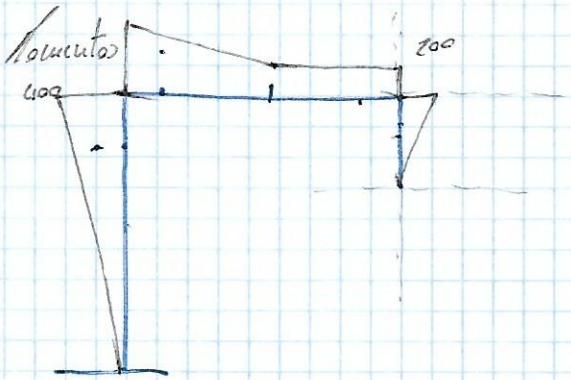
b) Determinar la magnitud de los movimientos horizon y vertical en el nudo S

$\Sigma F_x = 0 \quad A_x - 100 = 0 \rightarrow A_x = 100\ kN$

$\Sigma F_y = 0 \quad A_y - 50 = 0 \rightarrow A_y = 50\ kN$

$M_A \qquad M_A - 50 \cdot 4 + 100 \cdot 2 = 0 \rightarrow M_A = 0$

(M)



$M_B \qquad M_B + 100 \cdot 4 = 0$

$M_B = -400$

$M_C \qquad M_C + 100 \cdot 4 - 50 \cdot 8 + 50 \cdot 4 = 0$

$M_C = -200$

$M_D \qquad M_D + 100 \cdot 4 - 50 \cdot 4 = 0$

$M_D = 200$

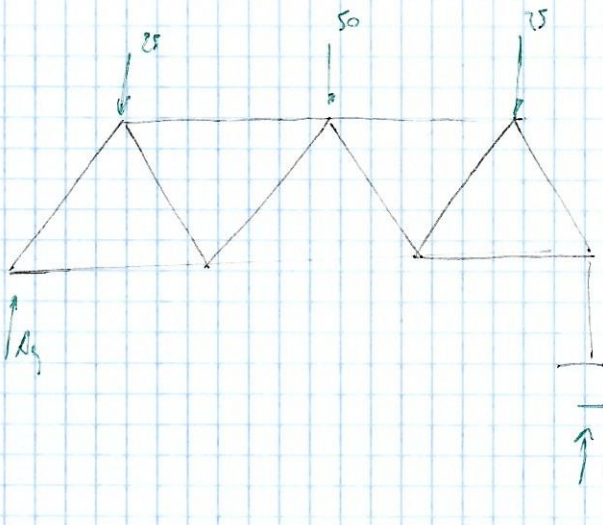
Método gráfico

$$u_{Dx} = \left(\frac{400 \cdot 4}{2} \cdot 8\right) + \left(200 \cdot 8 \cdot 4\right) + \left(\frac{200 \cdot 4}{2} \cdot \frac{6}{3} \cdot 4\right) = \frac{1360}{EI}$$

$$u_{Dy} = \left(\frac{400 \cdot 4}{2} \cdot 0'67\right) + \left(200 \cdot 8 \cdot 2\right) + \left(\frac{200 \cdot 4}{2} \cdot 8\right) + \left(\frac{200 \cdot 2}{2} \cdot 1'33\right) = \frac{4802}{EI}$$

$$\theta_{D-A} = \left(\frac{400 \cdot 4}{2}\right) + \left(200 \cdot 8\right) + \left(\frac{200 \cdot 4}{2}\right) + \left(\frac{200 \cdot 2}{2}\right) = \frac{3000}{EI}$$

Ej 6



Barra	N_i^S
1-2	$F^S/2$
2-3 / 3-4	$F^S/2\sqrt{3}$
2-5 / 5-6	$-F^S/\sqrt{3}$
3-5 / 4-6	$F^S/\sqrt{3}$
3-5 / 4-2	$-F^S/\sqrt{3}$
4-5	$\frac{\sqrt{3}}{2}F^S$

$$\bar{F}^S \cdot V_2 = \frac{\sum N_i \cdot N_i^S \cdot L}{EA} \qquad \text{Si } F^S = 1$$

$$V_3 = \frac{1}{EA}\left[\frac{50}{2}\cdot 2 + \frac{28'87}{2\sqrt{3}}\cdot 2(2) + \frac{57'74}{\sqrt{3}}\cdot 2\cdot 2 + \frac{28'87}{\sqrt{3}}\cdot 2\cdot 2 + \frac{57'74}{\sqrt{3}}\cdot(2)(2) + \frac{28'87}{\sqrt{3}}\cdot 2\cdot 2 + \frac{57'74}{2}\cdot 2\sqrt{3}\right]$$

$$V_3 = \frac{1}{200\cdot 10^6 \cdot A}\cdot\left[150 + \frac{57'74}{\sqrt{3}} + \frac{250'95}{\sqrt{3}} + \frac{115'48}{\sqrt{3}} + \frac{173'2}{\sqrt{3}} + \frac{115'48}{\sqrt{3}}\right]$$

$$V_3 = \frac{550}{EA}$$

Desplazamiento teniendo en cuenta $\Delta T = 50°C$

$$\bar{F}^S V_3 = \frac{\sum N_i \cdot N^S}{E\cdot A} L + \sum N_i^S \cdot \alpha \cdot \Delta T \cdot L_i$$

$$\sum N_i^S \alpha \Delta T L_i = \left(\frac{-1}{\sqrt{3}}\cdot \alpha \cdot 50 \cdot 2\right)\cdot 2 = \frac{-4\alpha\cdot 50}{\sqrt{3}} =$$

$$= -\alpha \cdot 115'47 \rightarrow \alpha = 11\cdot 10^{-6} \rightarrow V_3 = 1'27\ mm$$