Final Exam

Mathematical Methods of Bioengineering
Ingenería Biomédica - INGLÉS

14 of May 2019

The maximum time to make the exam is 3 hours. You are allowed to use a calculator and two sheets with annotations.

Problems

1. Consider the function $f(x, y, z) = (e^{xyz}, \tan yz, xy)$ and let $g : \mathbb{R}^3 \longrightarrow \mathbb{R}$ be a differentiable function that verifies $\nabla g(1, 1, 0) = (1, 1, -1)$.

   (a) (**1 point**) If $F = g \circ f$, compute $\nabla F(0, \frac{\pi}{4}, 1)$.

   (b) (**1 point**) Find the tangent plane of $F = 0$ at $(0, \frac{\pi}{4}, 1)$.

2. A laboratory that designs **nasogastric tubes** decides to model a tube of a *Levine catheter*. Assume that the thickness of the tube is one and that can be modelled with the parametric equations:

$$\mathbf{x}(t) = \left(\sqrt{2} \cdot t, e^t, e^{-t}\right), \quad 0 \leq t \leq 1 \ .$$

   (a) (**1 point**) Compute the length of the catheter tube.

   (b) (**1 point**) Suppose that through the tube is going to be administrated a medicament with density:

$$f(x, y, z) = \frac{1}{y^2 z^2}$$

   Compute the total quantity of medicament that can be accumulated inside the catheter tube.

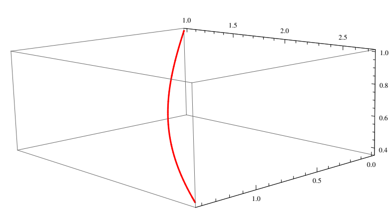

Figure 1: Catheter tube.

Observation: $(e^t + e^{-t})^2 = e^{2t} + e^{-2t} + 2$

3. Consider the surface shown in figure 2 and the region $D_{r_0} = \{(x, y) : x^2 + y^2 \leq r_0^2\}$ where r_0 is a constant value.

   (a) (**1 point**) Find the volume under the surface over the region D_{r_0}.

   (b) (**0.5 points**) Find the volume under the surface over all $\mathbb{R}^2$.

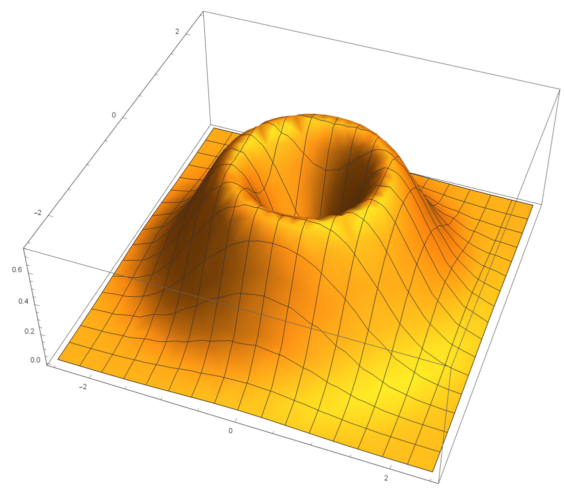

Figure 2: $f(x, y) = 2(x^2 + y^2)e^{-x^2-y^2}$.

Observation: $\lim_{r_0 \to \infty} D_{r_0} = \mathbb{R}^2$.

Integration by parts: $\int u \cdot v' dx = u \cdot v - \int u' \cdot v \, dx$

4. Suppose that you go every day to work by subway. You walk to the same subway station, which is served by two subway lines, both stopping near where you work. Each subway line sends trains to arrive at the stop every 6 minutes, but the dispatchers (train drivers) begin the schedules at random times.

   (a) (**0.5 points**) Given the arrival time x minutes for the first line train, and y minutes for the second line train, which is the function $T(x, y)$ that give the waited time?

   (b) (**1 point**) What is the average time you expect to wait for a subway train?

   Note: You could model the waiting time for the two subway lines by using a point (x, y) in the square $[0, 6] \times [0, 6]$.

   Note: The average of function T over a region $D \subset \mathbb{R}^2$ is $T_{avg} = \dfrac{1}{A(D)} \displaystyle\int\int_D T(x, y) \, dxdy$.

5. Consider the force field given by $\mathbf{F}(x, y) = (x + y, x - y)$.

   (a) (**1.5 points**) Prove that $\mathbf{F}$ is conservative and compute its potential function $f(x, y)$.

   (b) (**1 point**) Compute the line integral of $\mathbf{F}$ over the parabola $y = x^2$ with $x \in [0, 1]$.

   (c) (**0.5 points**) Compute the line integral of $\mathbf{F}$ over the path delimited by the parabola $y = x^2$ and the line $y = 1$ with counterclockwise orientation.