# Retake Exam

Mathematical Methods of Bioengineering
Ingeniería Biomédica - INGLÉS

21 of June 2019

*The maximum time to make the exam is 3 hours. You are allowed to use a calculator and two sheets with annotations. IMPORTANT: Question **2 A** is only for "alumnos nuevo ingreso" and question **2 B** is only for "alumnos veteranos".*

## Problems

1. Consider the surface $z = x^2 - 6x + y^3$.

   (a) (**1 point**) Find the tangent plane at the origin.

   (b) (**1 point**) Find the point/s on the surface, where the tangent plane are parallel to the plane $\pi : 4x - 12y + z = 7$.

2. **A**. Consider the function $f(x, y) = \dfrac{\sin \pi x}{1 + y^2}$.

   (a) (**1 point**) Find the critical points of $f$.

   (b) (**1 point**) Find the extrema nature of the critical point $(\frac{1}{2}, 0)$.

   **B**. A engineering is working with two mechanical arms with movements in a plane. To make a labor minimising the effort, he found that the optimal trajectories of the arm hands are $\mathbf{m_1}(t) = (t^2 - 2, \frac{t^2}{2} - 1)$ and $\mathbf{m_2}(t) = (t, 5 - t^2)$, where $t$ represents time measured in seconds. Before running the experiment, he simulated the trajectories and found out that collide.

   (a) (**1 point**) When and where do the arms collide?

   (b) (**1 point**) What is the angle formed by the paths of the arms at the collision point?

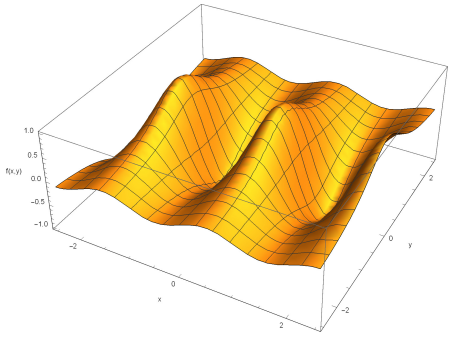


Figure 1: Function $f(x, y) = \dfrac{\sin \pi x}{1 + y^2}$ from 2 **A**.

3. (**2 points**) Suppose you are working doing artificial organs/tissues and you have a method to print in 3D materials with varying density. For a first test you decide to print a tissues that is modelled as the solid bounden by the surface $z = 1 - x^2$ and the planes $z = 0$, $y = 1$ and $y = -1$, as shown in the next figure.

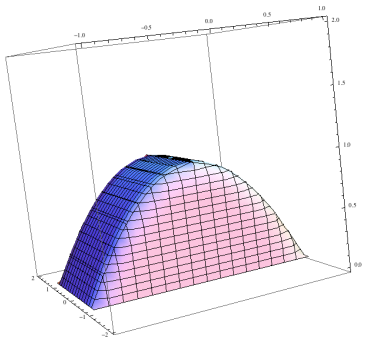

Figure 2: Artificial tissue.

Suppose also that the density is varying according to the function $g(x, y, z) = z(x + 2)$. Compute the total mass of the tissue.

4. (**2 points**) Evaluate $\oint_C (x^4y^5 - 2y)dx + (3x + x^5y^4)dy$, where $C$ is the oriented curve pictured below.

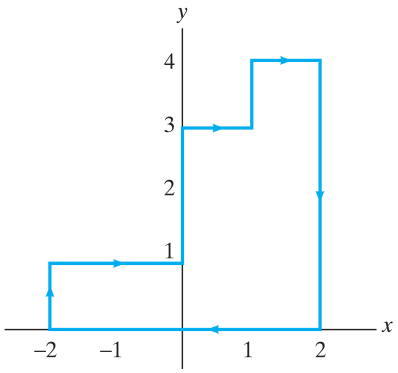


Figure 3: Oriented curve $C$.

5. Consider the curve $r(t) = (\cos t, \sin t, (a\cos t + b\sin t))$ with $0 \leq t \leq 2\pi$, where $a, b \in \mathbb{R}$ are constants.

   (a) (**1 point**) Compute the work done by the vector field $\mathbf{F} = (y, z - x, -y)$ in a particle moving along $r$.

   (b) (**1 point**) Compute the values of $a, b$ such that the work done is null. Does this mean that the vector field is conservative? Reasonate the answer.