Midterm Exam I. Page 2 of 2.

The terms of this examination formulated on page 1 also apply to page 2.

Problem 4. (*1 mark*) Study the continuity of the following function in $x = 0$. If the function is discontinuous, state the type of discontinuity.

$$f(x) = \begin{cases} \dfrac{e^{-x} - e^{x}}{x} & \text{for } x \neq 0 \\ 2 & \text{for } x = 0 \end{cases}$$

Problem 5. (*1.5 marks*) By applying the definition of the derivative, determine the derivative of

$$f(x) = \frac{x}{x+1}$$

in a point $x = a$. Give the domains of f and f'.

Problem 6. (*1 mark*) Consider the function

$$f(x) = (x+1)(x-1)^{2/3} - 3/2$$

on the interval $[0, 3]$. Study (a) the continuity of f and (b) the differentiability of f.

Remark: (a) is worth 0.5 marks, (b) is worth 0.5 marks.

Problem 7. (*1.5 marks*) Consider the function

$$f(x) = (1-x)^3$$

on the interval $[-2, 2]$. Justify the applicability of the mean value theorem and find the value(s) of c (from the theorem) satisfying the theorem.

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4. $f(x) = \begin{cases} \dfrac{e^{-x} - e^{x}}{x} & \text{for } x \neq 0 \\ 2 & \text{for } x = 0 \end{cases}$

$$\lim_{x \to 0^{\pm}} \frac{e^{-x} - e^{x}}{x} = \left[\frac{0}{0}\right] \overset{\text{L'Hôpital}}{=} \lim_{x \to 0^{\pm}} \frac{-e^{-x} - e^{x}}{1} = -2 = L$$

$$f(0) = 2 \neq L$$

$\Rightarrow f$ has a removable/evitable discontinuity at $x = 0$

5. $f(x) = \dfrac{x}{x+1}$ $\quad dom(f) = \mathbb{R} \setminus \{-1\}$

$$f'(a) \overset{\text{Def}}{=} \lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{a+\Delta x}{a+\Delta x+1} - \frac{a}{a+1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left(\frac{(a+\Delta x)(a+1) - a(a+\Delta x+1)}{(a+\Delta x+1)(a+1)} \right)$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left(\frac{a^2 + a + a\Delta x + \Delta x - a^2 - a\Delta x - a}{(a+\Delta x+1)(a+1)} \right)$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left(\frac{\Delta x}{(a+\Delta x+1)(a+1)} \right) = \lim_{\Delta x \to 0} \frac{1}{(a+\Delta x+1)(a+1)} = \frac{1}{(a+1)^2}$$

$dom(f') = \mathbb{R} \setminus \{-1\}$ since $f'(x) = \dfrac{1}{(x+1)^2}$

(f is not defined in $x = -1$, and hence neither

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6. $f(x) = (x+1)(x-1)^{2/3} - \frac{3}{2}$

a) $(x-1)^2$, $(x+1)$ and $(-\frac{3}{2})$ are polynomials of degree 2, 1 and 0 and as such continuous in $\mathbb{R}$.
The root function $x^{\frac{1}{3}}$ (cube root) is continuous in $\mathbb{R}$.
Sums and products of continuous functions are continuous in their domains (here, in $\mathbb{R}$), and hence in $[0,3]$, as well as the composition $(x-1)^{2/3}$.

b) $f'(x) = (x+1)\cdot\frac{2}{3}(x-1)^{-\frac{1}{3}} + (x-1)^{2/3}\cdot 1$

$$= \frac{2(x+1)}{3(x-1)^{\frac{1}{3}}} + \frac{(x-1)^{2/3}\cdot 3(x-1)^{\frac{1}{3}}}{3(x-1)^{\frac{1}{3}}} = \frac{5x-1}{3(x-1)^{1/3}}$$

$\Rightarrow f$ is not differentiable in $x=1$, but in the rest of the interval it is.

7. $f(x) = (1-x)^3 \qquad dom(f) = [-2,2] = I$

f is a polynomial (of order 3) and as such continuous and differentiable in $\mathbb{R}$ (and hence in I). Therefore, the MVT can be applied: $\exists\, c \in (-2,2)$ with

$$f'(c) = \frac{f(b)-f(a)}{b-a} \quad \text{with } a=-2,\ b=2$$

$$\frac{(-1)^3 - 3^3}{4} = \frac{-1-27}{4} = -7$$

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3. [illegible]

And $c = \frac{1}{3}$ is the only solution $\in (-2,2)$

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