INTEGRAL CALCULUS. SECOND MIDTERM

In the following problems you are required to show all your work and provide the necessary explanations to get full credit. Only scientific calculators with no graphing capabilities are allowed.

1. (3.25 POINTS) Compute the volume of the solid

$$A = \{(x, y, z) \in \mathbb{R}^3 : \ 2x^2 + y^2 \leq z \leq 2 - x^2 - y^2\}.$$

2. (3.25 POINTS) Evaluate the integral $\displaystyle\int_B x^2y^2z \, dxdydz$, where

$$B = \{(x, y, z) \in \mathbb{R}^3 : \ x^2 + y^2 \leq z^2 \leq 3x^2 + 3y^2; \ x^2 + y^2 + z^2 \leq 1; \ z \geq 0\}.$$

3. (3.5 POINTS) Evaluate the integral $\int_R e^{x-3y} \cos^2(\pi(x+y))dxdy$, where R is the parallelogram with vertices $(1, 0)$, $(4, 1)$, $(\frac{7}{4}, \frac{-3}{4})$ and $(\frac{19}{4}, \frac{1}{4})$ by making a change of variables $(x, y) = \varphi(u, v)$ that maps a rectangle S in the uv-plane (where the sides of S are parallel to the u- and v-axes) onto the parallelogram R in the xy-plane.