

Ordering

The set of real numbers is equipped with an order relation, denoted by $<$, such that for any given two real numbers a and b , there are three mutually exclusive possibilities:

(i) $a < b$ (a is less than b)

(ii) $a = b$ (a equals b)

(iii) $b < a$ (b is less than a)

$$b > a$$

We define

(iv) $a \leq b$ (a is less than or equal to b)

(v) $a \geq b$ (a is greater than or equal to b).

The logo for Cartagena99 features the text 'Cartagena99' in a stylized, bold font. The '99' is significantly larger and more prominent than the 'Cartagena' part. The text is set against a background of overlapping light blue and orange shapes.

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For every $x, y, z \in \mathbb{R}$, the following properties hold:

Transitivity:

If $x < y$ and $y < z$, then $x < z$.

Compatibility with addition:

If $x < y$, then $x + z < y + z$.

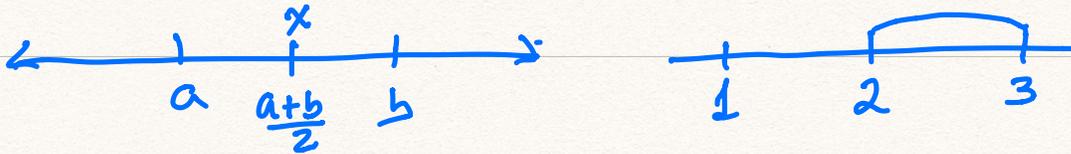
Multiplication by a positive factor:

If $x < y$ and $0 < z$, then $xz < yz$.

Multiplication by a negative factor:

If $x < y$ and $z < 0$, then $xz > yz$.

Example: Suppose that $a < b$. There exists a real number x satisfying $a < x < b$.



We will prove that $x = \frac{a+b}{2}$ satisfies $a < x < b$.

$$a < b \Rightarrow 2a < a+b \Rightarrow a < \frac{a+b}{2} \quad (1)$$

\uparrow add a to both sides \uparrow divide by 2

$$a < b \Rightarrow a+b < 2b \Rightarrow a+b < b \quad (2)$$

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Example: If $b > 0$ and $B > 0$ and

$$\frac{a}{b} < \frac{A}{B} \implies \frac{a \cdot B}{b \cdot B} < \frac{A \cdot b}{B \cdot b} \implies aB < Ab$$

mult. both sides by bB

then $aB < bA$. Deduce that

$$\frac{a}{b} < \frac{a+A}{b+B} < \frac{A}{B}$$

• $\frac{a}{b} < \frac{a+A}{b+B}$:

~~$a(b+B) < (a+A)b$
 $ab + aB < ab + Ab$
 $aB < Ab$~~

Adding ab to both sides of $aB < Ab$ we get

$$ab + aB < Ab + ab \implies a(b+B) < (A+a)b \implies$$

\downarrow
 $b+B > 0$
 $b > 0$

$$\frac{a(b+B)}{b(b+B)} < \frac{(A+a)b}{b(b+B)} \implies \frac{a}{b} < \frac{A+a}{b+B}$$

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Example: If $\overset{!}{a} > 0$, then $\overset{!}{a^{-1}} > 0$.
Always true Prove this is true.

Assume that $a > 0$ and $a^{-1} \leq 0$.

$a^{-1} < 0$ or $\cancel{a^{-1} = 0}$.
Case II Case I

Case I:

Suppose that $a^{-1} = 0$. We know that $a \cdot a^{-1} = 1$.

But $a^{-1} = 0$ so $a \cdot a^{-1} = 0 \Rightarrow 0 = 1$ Contradiction!

So $a^{-1} \neq 0$.

\rightarrow state 1
false \rightarrow state 2 } Contradiction

Case II:

Now suppose that $a^{-1} < 0$. Multiplying both

sides by $a > 0$, we get $a \cdot a^{-1} < a \cdot 0$ or

$$1 = a \cdot a^{-1} < a \cdot 0 = 0 \Rightarrow 1 < 0.$$

So $\overset{!}{a^{-1}} > 0$.

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Example: If x and y are positive, then $x < y$ if and only if $x^2 < y^2$.

• If $x < y$ then $x^2 < y^2 \implies$

• If $x^2 < y^2$ then $x < y \Leftarrow$

• If $x < y$ then $x^2 < y^2$

Multiply both sides of $x < y$ times x we get $x^2 < xy$.

Multiply both sides of $x < y$ times y we get $xy < y^2$

By transitivity $x^2 < y^2$.

• If $x^2 < y^2$ then $x < y$

$$\begin{cases} x < y & x+y > 0 \\ x-y < 0 \end{cases}$$

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example, $(x+y)^{-1} > 0$. Multiplying
both sides of (1) by $(x+y)^{-1}$ we get

$$x-y < 0 \Rightarrow x < y.$$

add
y to both
sides

$$\cancel{(x+y)^{-1}} \cancel{(x+y)} (x-y) < 0 \quad \cancel{(x+y)^{-1}} = 0$$

$$x-y < 0$$

$$a = x+y > 0$$

$$a^{-1} = (x+y)^{-1} > 0$$

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