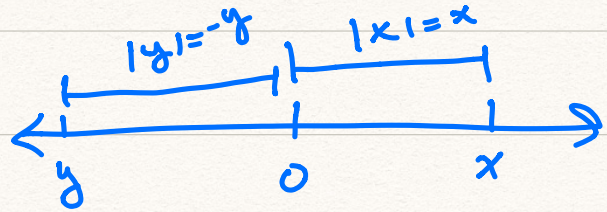


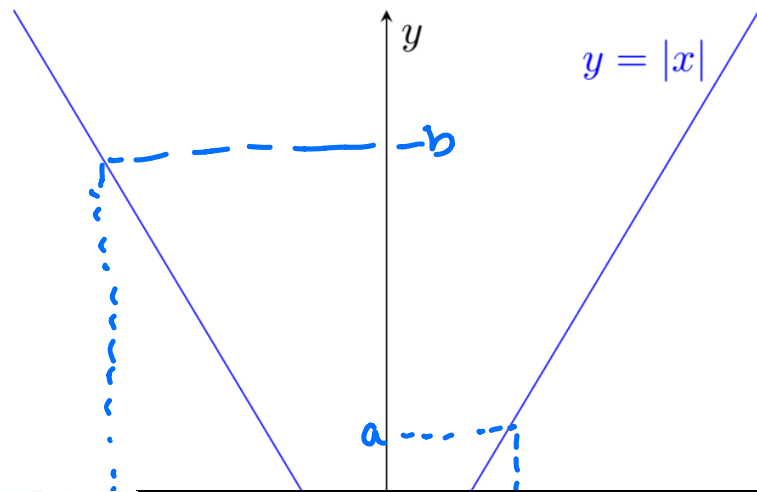
Let  $x$  be a real number. Its modulus (or absolute value)  $|x|$  is defined by

$$|x| = \begin{cases} x & (x \geq 0) \\ -x & (x < 0) \end{cases}$$



It is sometimes useful to note that

$$|x| = \sqrt{x^2} \neq (\sqrt{x})^2$$



$$|x| \geq 0$$

$$|x| = 0$$

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## Theorem

For any real number  $x$ ,  $-|x| \leq x \leq |x|$ .

Proof:

If  $x \geq 0$ , then  $-|x| \leq 0 \leq x = |x|$   
 $-|x| \leq x \leq |x|$

If  $x < 0$ , then  $-|x| = x < 0 \leq |x|$   
 $-|x| \leq x \leq |x|$

## Theorem

For any real numbers  $a$  and  $b$ ,  $|ab| = |a||b|$ .

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## Theorem: Triangle Inequality

For any real numbers  $a$  and  $b$ ,

$$|a+b| \leq |a| + |b|$$

Proof:

$$|a+b| = \sqrt{(a+b)^2}$$

$$|a+b|^2 \stackrel{\downarrow}{=} (a+b)^2$$

$$ab \leq |ab|$$

$$= a^2 + 2ab + b^2$$

$$= |a|^2 + 2ab + |b|^2$$

$$\leq |a|^2 + 2|ab| + |b|^2$$

$$= |a|^2 + 2|a||b| + |b|^2$$

$$= (|a| + |b|)^2$$

$$|a+b|^2 \leq (|a| + |b|)^2 \Rightarrow |a+b| \leq |a| + |b|$$

Remark:  $|x|=0$  if and only if  $x=0$ .

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**Problem 1.2** Prove that  $|a+b|^2 = (|a|+|b|)^2$  if and only if  $ab \geq 0$

$$|x| = \sqrt{x^2}$$



$$\Leftarrow |a+b|^2 = (a+b)^2$$

$$= a^2 + 2ab + b^2$$

$$= |a|^2 + 2ab + |b|^2$$

$$|x| = x$$

$$= |a|^2 + 2|a||b| + |b|^2$$

$$|ab| = |a||b|$$

$$= |a|^2 + 2|a||b| + |b|^2$$

$$= (|a| + |b|)^2$$

If  $ab \geq 0$  then  $|a+b| = |a| + |b|$

$$\Rightarrow |a+b|^2 = (a+b)^2$$

$$|a+b|^2 = (|a| + |b|)^2$$

$$a^2 + 2ab + b^2 = |a|^2 + 2|a||b| + |b|^2$$

$$|a|^2 + 2ab + |b|^2 = |a|^2 + 2|a||b| + |b|^2$$

$$2ab = 2|a||b| \geq 0$$

$$|x| \geq 0$$

$$ab \geq 0$$

$$2ab \geq 0$$

$$a^2 + 2ab + b^2 \geq a^2 + b^2$$

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$$a = -3 \quad b = -2$$

$$ab = (-3)(-2) = 6$$

$$|a||b| = |-3||-2| = 3 \cdot 2 = 6$$

The logo for Cartagena99 features the text 'Cartagena99' in a stylized, teal-colored font. The '99' is significantly larger and more prominent than the 'Cartagena' part. The text is set against a background of light blue and orange geometric shapes, including a large blue triangle and an orange shape below it.

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