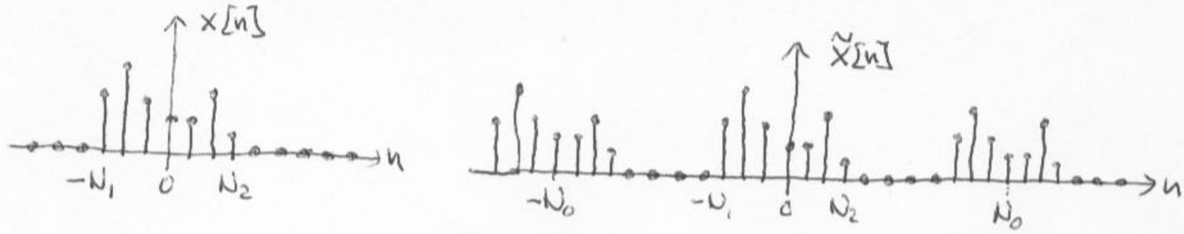
TEMA 5: TRANSFORMADA DE FOURIER DE SEÑALES DE TIEMPO DISCRETO (DTFT)

* INTRODUCCIÓN

* DTFT DE SEÑALES APERIÓDICAS

· EXPRESIÓN GENERAL

- DADA UNA SEÑAL APERIÓDICA $x[n]$ DE DURACIÓN FINITA



$$\tilde{x}[n] = \sum_{k=\langle N_0\rangle} a_k e^{jk\omega_0 n} \xrightarrow{\text{DTFS}} a_k = \frac{1}{N_0}\cdot\sum_{n=\langle N_0\rangle}\tilde{x}[n]e^{-jk\omega_0 n} =$$

$$= \frac{1}{N_0}\cdot\sum_{n=-\infty}^{\infty}x[n]e^{-jk\omega_0 n} \Rightarrow a_k = \frac{1}{N_0}\cdot X(e^{j\omega})\Big|_{\omega=k\omega_0}, \text{ siendo } \boxed{X(e^{j\omega}) = \sum_{n=-\infty}^{\infty}x[n]e^{-j\omega n}},$$

QUE ES CONTINUA EN ω Y PERIÓDICA DE PERIODO 2π

· PARA SINTETIZAR O RECUPERAR $x[n]$ A PARTIR DE $X(e^{j\omega})$:

$$\tilde{x}[n] = \sum_{k=\langle N_0\rangle} a_k e^{jk\omega_0 n} = \sum_{k=\langle N_0\rangle}\cdot\frac{1}{N_0}X(e^{jk\omega_0})\cdot e^{jk\omega_0 n} \underset{\omega_0 = 2\pi/N_0}{=} \frac{1}{2\pi}\cdot\sum_{k=\langle N_0\rangle}X(e^{jk\omega_0})e^{jk\omega_0 n}\cdot\omega_0$$

$$x[n] = \lim_{N_0\to\infty}\tilde{x}[n] = \lim_{\omega_0\to 0}\frac{1}{2\pi}\cdot\sum_{k=\langle N_0\rangle}X(e^{jk\omega_0})\cdot e^{jk\omega_0 n}\omega_0 \Rightarrow$$

$\sum_{k=\langle N_0\rangle} \rightarrow$ N_0 VALORES SEPARADOS $2\pi/N_0$ ENTRE SÍ.

$$\Rightarrow \boxed{x[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle}X(e^{j\omega})\cdot e^{j\omega n}d\omega}$$

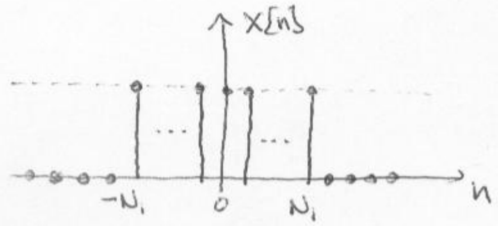
· PERIODICIDAD DE $X(e^{j\omega})$:

$$X(e^{j(\omega+2k\pi)}) = \sum_{n=-\infty}^{\infty}x[n]e^{-j(\omega+2k\pi)n} = \sum_{n=-\infty}^{\infty}x[n]e^{-j\omega n} = X(e^{j\omega}) \Rightarrow$$ SU PERIODO

FUNDAMENTAL ES 2π.

• EJEMPLOS DE CÁLCULO

(A)



$$x[n] \xrightarrow{\text{DTFT}} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} = \sum_{n=-N_1}^{N_1} e^{-j\omega n} = \frac{e^{j\omega N_1} - e^{-j\omega(N_1+1)}}{1-e^{-j\omega}} =$$

$$= \frac{e^{-j\frac{\omega}{2}}}{e^{-j\frac{\omega}{2}}} \cdot \frac{e^{j\omega(N_1+\frac{1}{2})} - e^{-j\omega(N_1+\frac{1}{2})}}{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}} = \frac{\operatorname{sen}\left(\omega\left(N_1+\frac{1}{2}\right)\right)}{\operatorname{sen}(\omega/2)}, \quad \omega \neq 0$$

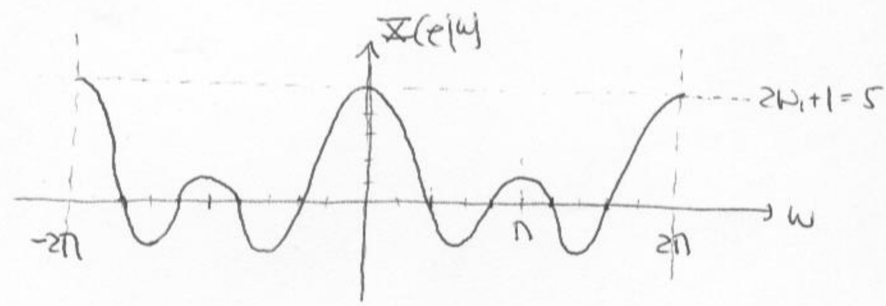
PARA $\omega = 0$, $X(e^{j0}) = \sum_{n=-\infty}^{\infty} x[n] = 2N_1+1$

REPRESENTACIÓN:

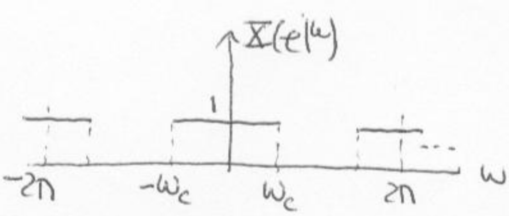
$X(e^{j\omega})$ PERIÓDICA 2π, CON CEROS EN $\omega\left(N_1+\frac{1}{2}\right) = k\pi \Rightarrow \omega = \frac{2k\pi}{2N_1+1} \Rightarrow$

ENTRE 0 Y 2π TIENE $2N_1$ CEROS

EJEMPLO: PARA $N_1 = 2$, CEROS EN $\omega = 2k\pi/5$



(B)

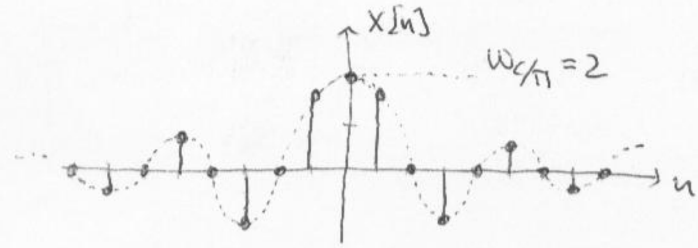


$$\xrightarrow{\text{DTFT}^{-1}} x[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega})\, e^{j\omega n}\, d\omega =$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n}\, d\omega = \frac{1}{2\pi}\, \frac{e^{j\omega_c n} - e^{-j\omega_c n}}{jn} = \frac{\operatorname{sen}(\omega_c n)}{\pi n} = \frac{\omega_c}{\pi}\operatorname{sinc}\left(\frac{\omega_c n}{\pi}\right)$$

, que es una 'sinc' con ceros en $\frac{k\pi}{\omega_c}$

Ejemplo: para $\omega_c = \pi/2$, ceros en $2k$:



- Convergencia de la DTFT

  - La expresión de síntesis, $x[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j\omega n}d\omega$, no presenta problemas de convergencia al extenderse la integral a un intervalo finito.

  - La expresión de análisis, $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$, converge si:

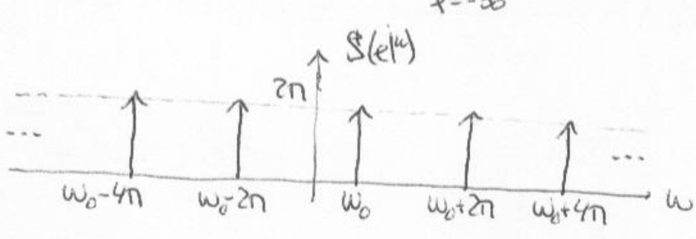
$$\sum_{n=-\infty}^{\infty}|x[n]| < \infty \quad ó \quad \sum_{n=-\infty}^{\infty}|x[n]|^2 < \infty$$

* DTFT de señales periódicas

· En tiempo continuo $e^{j\omega_0 t} \xrightarrow{FT} 2\pi\delta(\omega-\omega_0)$

· En tiempo discreto cabe esperar algo similar, pero periódico 2π:

$$e^{j\omega_0 n} \xrightarrow{DTFT} \sum_{l=-\infty}^{\infty} 2\pi\delta(\omega-\omega_0-2\pi l) = S(e^{j\omega})$$



$$a_k = a_{k+N_0}$$
$$\downarrow \qquad \downarrow$$
$$k\omega_0 \qquad k\omega_0+2\pi$$

Comprobamos que:

$$\frac{1}{2\pi}\int_{\langle 2\pi\rangle} S(e^{j\omega})e^{j\omega n}d\omega = e^{j(\omega_0+2\pi l)n} = e^{j\omega_0 n}$$

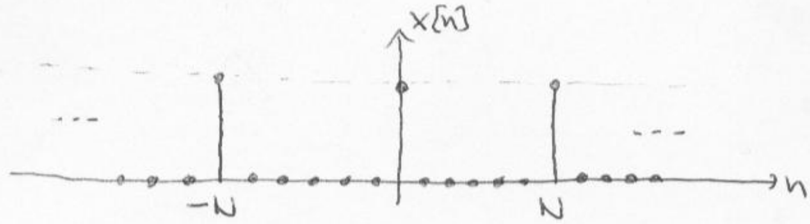
Por lo tanto:

$$\boxed{x[n] = \sum_{k=\langle N_0\rangle} a_k e^{jk\omega_0 n} \xrightarrow{DTFT} X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi a_k \cdot \delta(\omega-k\omega_0)}$$

EJEMPLO:

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-kN] \xrightarrow{DTFS} a_k = \frac{1}{N} \cdot \sum_{n=\langle N\rangle} x[n] e^{-jk\omega_0 n} =$$

$$= \frac{1}{N}, \ \forall k$$



$$x[n] \xrightarrow{DTFT} X(e^{j\omega}) = \frac{2\pi}{N} \cdot \sum_{k=-\infty}^{\infty} \delta(\omega - k\frac{2\pi}{N})$$

* PROPIEDADES DE LA DTFT:

① LINEALIDAD

② DESPLAZAMIENTOS

$$x[n] \xrightarrow{DTFT} X(e^{j\omega})$$

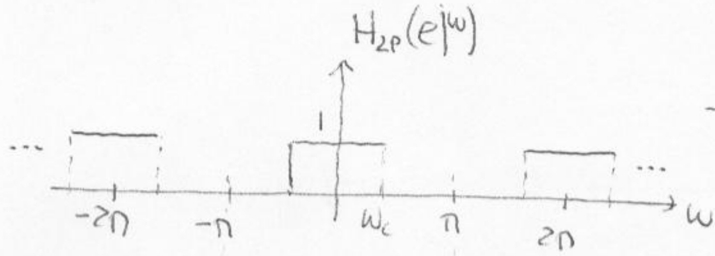
a) En 't': $\quad x[n-n_0] \xrightarrow{DTFT} e^{-j\omega n_0} \cdot X(e^{j\omega})$

b) En 'ω': $\quad e^{j\omega_0 n} \cdot x[n] \longleftarrow X(e^{j(\omega-\omega_0)})$

③ CONJUGACIÓN Y SIMETRÍAS

$$\left.\begin{array}{l} x[n] \xrightarrow{DTFT} X(e^{j\omega}) \\ x^*[n] \xrightarrow{DTFT} X^*(e^{-j\omega}) \\ x[-n] \xrightarrow{DTFT} X(e^{-j\omega}) \end{array}\right\} \Rightarrow \text{PROPIEDADES ANÁLOGAS A FT.}$$

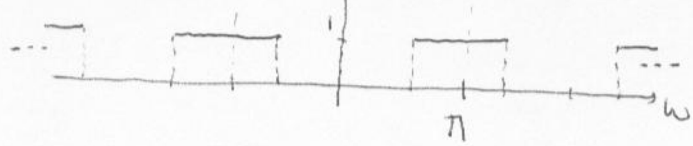
Ejemplo:



$$\xrightarrow{DTFT^{-1}} h_{LP}[n]$$

$$H_{BP}(e^{j\omega}) = H_{LP}(e^{j(\omega-\pi)}) \xrightarrow{DTFT^{-1}} h_{BP}[n] = e^{j\pi n} h_{LP}[n] =$$

$$= (-1)^n \cdot h_{LP}[n]$$



④ ESCALADO

$$\text{SEA } x_m[n] = \begin{cases} x[n/m] & , n \text{ MÚLTIPLO DE } m \\ 0 & , \text{RESTO} \end{cases} \quad , m \in \mathbb{Z}^+$$

$$X_m(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_m[n] e^{-j\omega n} \underset{\substack{\downarrow \\ n=\ell m}}{=} \sum_{\ell=-\infty}^{\infty} x[\ell] e^{-j\omega \ell m} = X(e^{j\omega m})$$

⑤ PRIMERA DIFERENCIA Y SUMA ACUMULADA

$$x[n] \xrightarrow{DTFT} X(e^{j\omega})$$

$$x[n]-x[n-1] \xrightarrow{DTFT} (1-e^{-j\omega}) \cdot X(e^{j\omega})$$

$$n \cdot x[n] \xleftarrow{DTFT^{-1}} j\frac{dX(e^{j\omega})}{d\omega}$$

$$y[n] = \sum_{k=-\infty}^{n} x[k] \xrightarrow{DTFT} \frac{X(e^{j\omega})}{1-e^{-j\omega}} + \pi X(e^{j0}) \cdot \sum_{k=-\infty}^{\infty} \delta(\omega - 2k\pi)$$

⑥ RELACIÓN DE PARSEVAL

$$\underbrace{\sum_{n=-\infty}^{\infty} |x[n]|^2}_{\text{ENERGÍA DE } x[n]} = \underbrace{\frac{1}{2\pi}\int_{\langle 2\pi \rangle} |X(e^{j\omega})|^2 d\omega}_{\text{POTENCIA O ENERGÍA POR PERIODO DE } X(e^{j\omega})}$$

* DUALIDAD

	TIEMPO CONTINUO	TIEMPO DISCRETO
SERIE DE FOURIER	$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$x[n] = \sum_{k=\langle N_0\rangle} a_k e^{jk\omega_0 n}$
	$a_k = \frac{1}{T_0}\int_{\langle T_0\rangle} x(t) e^{-jk\omega_0 t} dt$	② $a_k = \frac{1}{N_0}\cdot \sum_{n=\langle N_0\rangle} x[n] e^{-jk\omega_0 n}$
TRANSFORMADA DE FOURIER	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$	③ $x[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega}) e^{j\omega n} d\omega$
	① $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$	④ $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

---

① $x(t) \xrightarrow{FT} X(j\omega) \Rightarrow 2\pi x(-t) = \int_{-\infty}^{\infty} X(j\omega) e^{-j\omega t} d\omega \Rightarrow$

$\Rightarrow X(jt) \xrightarrow{FT} 2\pi x(-\omega)$

② $x[n] \xrightarrow{DTFS} a_k \Rightarrow \frac{1}{N_0} x[-n] = \frac{1}{N_0}\cdot \sum_{k=\langle N_0\rangle} a_k e^{-jk\omega_0 n} \Rightarrow$

$\Rightarrow a[n] \xrightarrow{DTFS} \frac{1}{N_0} x_{-k}$

$\omega_0 = \frac{2\pi}{T_0} = 1$

③ $x[n] \xrightarrow{DTFT} X(e^{j\omega}) \Rightarrow x[-n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega}) e^{-j\omega n} d\omega \Rightarrow$

$\Rightarrow \underbrace{X(e^{jt})}_{} \xrightarrow{FS} x_{-k}$

PERIÓDICA 2π O DE CUALQUIER PERIODO, YA QUE $X(e^{jat}) \xrightarrow{FS} x_{-k}$ TAMBIÉN

$\omega_0 = \frac{2\pi}{T_0} = 1$

④ $x(t) \xrightarrow{FS} a_k \Rightarrow x(-t) = \sum_{k=-\infty}^{\infty} a_k e^{-jk\omega_0 t} \Rightarrow$

$\Rightarrow a[n] \xrightarrow{DTFT} x(-\omega)$

* DTFT y sistemas LTI

- La propiedad de convolución

① $x[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j\omega n}d\omega = \lim_{\omega_0\to 0}\frac{1}{2\pi}\sum_{k=\langle N_0\rangle} X(e^{jk\omega_0})e^{jk\omega_0 n}\cdot\omega_0$

② $e^{jk\omega_0 n} \xrightarrow{LTI} H(e^{jk\omega_0})e^{jk\omega_0 n}$, $H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$

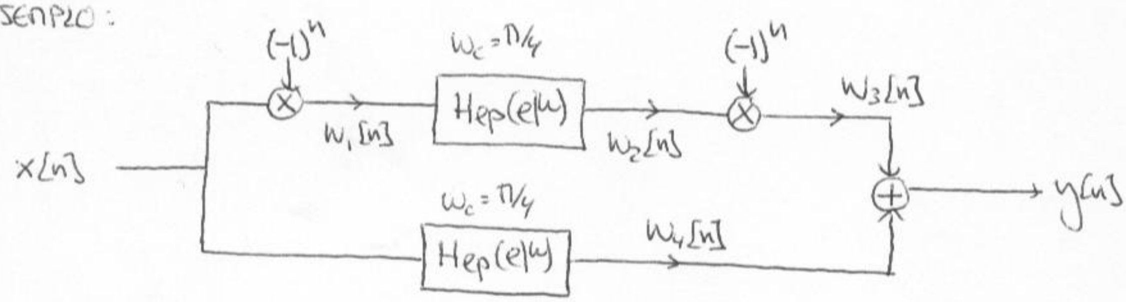
$\Rightarrow$

$$\Rightarrow x[n] \xrightarrow{LTI} y[n] = \lim_{\omega_0\to 0}\frac{1}{2\pi}\sum_{k=\langle N_0\rangle} X(e^{jk\omega_0})\cdot H(e^{jk\omega_0})\cdot e^{jk\omega_0 n}\cdot\omega_0 =$$

$$= \frac{1}{2\pi}\int_{\langle 2\pi\rangle} \underbrace{X(e^{j\omega})\cdot H(e^{j\omega})}_{Y(e^{j\omega})} e^{j\omega n}d\omega \Rightarrow$$

$$\Rightarrow y[n] = x[n] * h[n] \xrightarrow{DTFT} Y(e^{j\omega}) = X(e^{j\omega})\cdot H(e^{j\omega})$$

Ejemplo:



- $w_1[n] = (-1)^n\cdot x[n] = e^{j\pi n}x[n] \xrightarrow{DTFT} W_1(e^{j\omega}) = X(e^{j(\omega-\pi)})$
- $w_2[n] = w_1[n] * h_{lp}[n] \xrightarrow{DTFT} W_2(e^{j\omega}) = W_1(e^{j\omega})\cdot H_{lp}(e^{j\omega}) =$

$$= H_{lp}(e^{j\omega})\cdot X(e^{j(\omega-\pi)})$$

- $w_3[n] = (-1)^n w_2[n] \xrightarrow{DTFT} W_3(e^{j\omega}) = W_2(e^{j(\omega-\pi)}) = H_{lp}(e^{j(\omega-\pi)})\cdot \underbrace{X(e^{j(\omega-2\pi)})}_{= X(e^{j\omega})}$
- $w_4[n] = x[n] * h_{lp}[n] \xrightarrow{DTFT} W_4(e^{j\omega}) = H_{lp}(e^{j\omega})\cdot X(e^{j\omega})$

$$y[n] = w_3[n] + w_4[n] \xrightarrow{DTFT} Y(e^{j\omega}) = X(e^{j\omega})\underbrace{\left[H_{lp}(e^{j\omega}) + H_{lp}(e^{j(\omega-\pi)})\right]}_{\text{FILTRO DE BANDA ELIMINADA}}$$

- La propiedad de multiplicación

$$y[n] = r[n] \cdot s[n] \xrightarrow{\text{DTFT}} Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} r[n]\, s[n]\, e^{-j\omega n} =$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{1}{2\pi}\int_{\langle 2\pi\rangle} R(e^{j\theta})\, e^{j\theta n}\, d\theta\right] \cdot s[n]\, e^{-j\omega n} = \frac{1}{2\pi} \cdot \int_{\langle 2\pi\rangle} R(e^{j\theta}) \cdot \left[\sum_{n=-\infty}^{\infty} s[n]\, e^{-j(\omega-\theta)n}\right] d\theta =$$

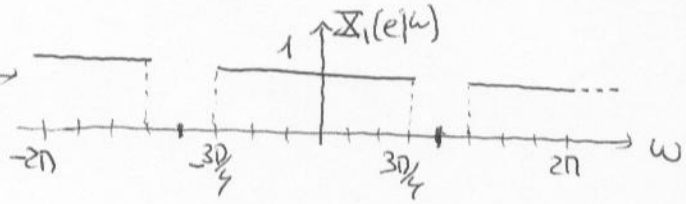
$$= \frac{1}{2\pi}\int_{\langle 2\pi\rangle} R(e^{j\theta}) \cdot S(e^{j(\omega-\theta)})\, d\theta = \frac{1}{2\pi} \cdot R(e^{j\omega}) \circledast S(e^{j\omega})$$

convolución periódica

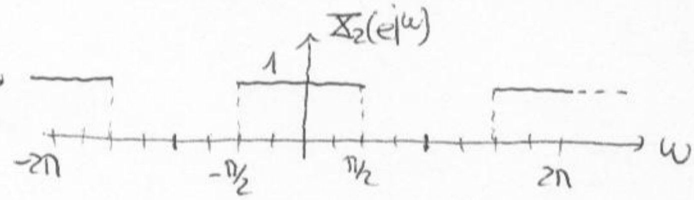
Ejemplo:

$$x[n] = x_1[n] \cdot x_2[n] \,, \quad x_1[n] = \frac{\operatorname{sen}\left(\frac{3\pi}{4}n\right)}{\pi n} \,, \quad x_2[n] = \frac{\operatorname{sen}\left(\frac{\pi}{2}n\right)}{\pi n}$$

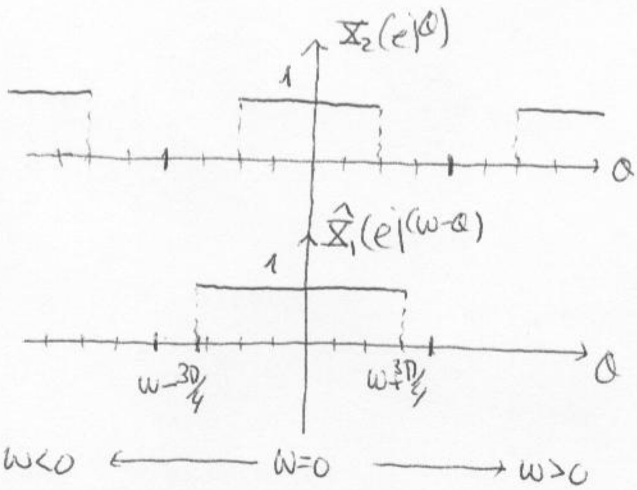
$x_1[n] \xrightarrow{\text{DTFT}}$

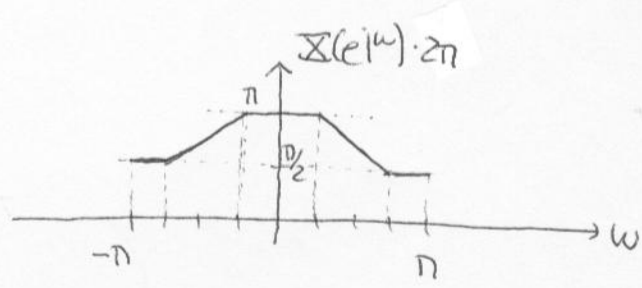


$x_2[n] \xrightarrow{\text{DTFT}}$



$$X(e^{j\omega}) = X_1(e^{j\omega}) \circledast X_2(e^{j\omega}) \cdot \frac{1}{2\pi} = \frac{1}{2\pi} \cdot \underbrace{\hat{X}_1(e^{j\omega})}_{\text{un periodo de } X_1(e^{j\omega})} * X_2(e^{j\omega})$$





* SISTEMAS DESCRITOS A PARTIR DE EDO's LINEARES CON COEFICIENTES CONSTANTES

$$\sum_{k=0}^{N} a_k\, y[n-k] = \sum_{k=0}^{M} b_k\, x[n-k] \xrightarrow{\text{DTFT}}$$

$$\longrightarrow \sum_{k=0}^{N} a_k e^{-jk\omega} \cdot Y(e^{j\omega}) = \sum_{k=0}^{M} b_k e^{-jk\omega} \cdot X(e^{j\omega}) \Rightarrow$$

$$\Rightarrow H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\sum_{k=0}^{M} b_k e^{-jk\omega}}{\sum_{k=0}^{N} a_k e^{-jk\omega}}$$

Ejemplo:

$$y[n] - a\,y[n-1] = x[n] \Rightarrow H(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}} \xrightarrow{\text{DTFT}^{-1}} h[n] = a^n u[n]:$$

$$h[n] = a^n u[n] \Rightarrow H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n} = \sum_{n=0}^{\infty} a^n e^{-j\omega n} = \sum_{n=0}^{\infty} (ae^{-j\omega})^n \underset{|a|<1}{=}$$

$$= \frac{1}{1-ae^{-j\omega}}$$

PROBLEMA 5.3

a) $x[n] = \operatorname{sen}\left(\frac{\pi}{3}n + \frac{\pi}{4}\right) = \frac{1}{2j} e^{j\left(\frac{\pi}{3}n+\frac{\pi}{4}\right)} - \frac{1}{2j} e^{-j\left(\frac{\pi}{3}n+\frac{\pi}{4}\right)} =$

$$= \underbrace{\frac{1}{2j} e^{j\frac{\pi}{4}}}_{a_1} \cdot e^{j\frac{\pi}{3}n} - \underbrace{\frac{1}{2j} e^{-j\frac{\pi}{4}}}_{a_{-1} = a_1^* \ (x[n] \text{ REAL})} \cdot e^{-j\frac{\pi}{3}n} \quad , \quad \omega_0 = \frac{\pi}{3}$$

$$X(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0) = \frac{\pi}{j} \left\{ e^{j\frac{\pi}{4}} \delta\left(\omega - \frac{\pi}{3}\right) - e^{-j\frac{\pi}{4}} \delta\left(\omega + \frac{\pi}{3}\right) \right\} \text{ PERIÓDICA } 2\pi$$

b) $x[n] = 2 + \cos\left(\frac{\pi}{6}n + \frac{\pi}{8}\right) = 2 + \frac{1}{2} e^{j\left(\frac{\pi}{6}n+\frac{\pi}{8}\right)} + \frac{1}{2} e^{-j\left(\frac{\pi}{6}n+\frac{\pi}{8}\right)} =$

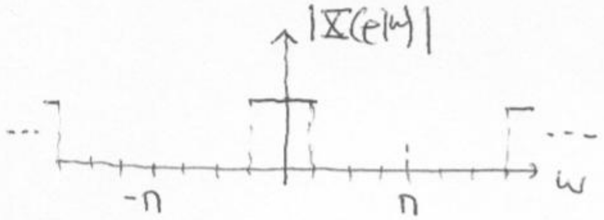
$$= \underbrace{2}_{a_0} + \underbrace{\frac{1}{2} e^{j\frac{\pi}{8}}}_{a_1} \cdot e^{j\frac{\pi}{6}n} + \underbrace{\frac{1}{2} e^{-j\frac{\pi}{8}}}_{a_{-1} = a_1^* \ (x[n] \text{ REAL})} e^{-j\frac{\pi}{6}n} \quad , \quad \omega_0 = \frac{\pi}{6}$$

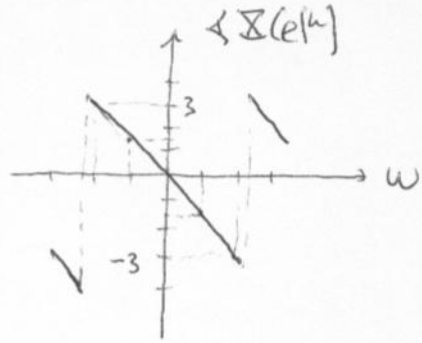
$$X(e^{j\omega}) = 4\pi \delta(\omega) + \pi e^{j\frac{\pi}{8}} \delta\left(\omega - \frac{\pi}{6}\right) + \pi e^{-j\frac{\pi}{8}} \delta\left(\omega + \frac{\pi}{6}\right)$$

PROBLEMA 5.5

DADA $X(e^{j\omega}) = |X(e^{j\omega})| e^{j \sphericalangle X(e^{j\omega})}$, OBTENGA $x[n]$ Y LOS VALORES DE n PARA LOS CUALES SE ANULA.

$$|X(e^{j\omega})| = \begin{cases} 1, & 0 \le |\omega| < \pi/4 \\ 0, & \pi/4 \le |\omega| \le \pi \end{cases}, \text{ PERIÓDICA } 2\pi \qquad \sphericalangle X(e^{j\omega}) = -\frac{3\omega}{2}$$





$$x[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega}) e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j\frac{3\omega}{2}} e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{j(n-\frac{3}{2})\omega} d\omega =$$

$$= \frac{1}{2\pi} \left. \frac{e^{j(n-\frac{3}{2})\omega}}{j(n-\frac{3}{2})} \right|_{-\pi/4}^{\pi/4} = \frac{1}{\pi(n-\frac{3}{2})} \cdot \frac{e^{j\frac{\pi}{4}(n-\frac{3}{2})} - e^{-j\frac{\pi}{4}(n-\frac{3}{2})}}{2j} =$$

$$= \frac{\operatorname{sen}\left(\frac{\pi}{4}\left(n-\frac{3}{2}\right)\right)}{\pi\left(n-\frac{3}{2}\right)} = \boxed{\frac{1}{4} \operatorname{sinc}\left(\frac{1}{4}\left(n-\frac{3}{2}\right)\right)}$$

$x[n] = 0$, PARA $\frac{1}{4}\left(n-\frac{3}{2}\right) = k \Rightarrow n = \frac{k}{4} + \frac{3}{2} \notin \mathbb{Z} \Rightarrow$

$\Rightarrow x[n] = 0,\ n = \pm\infty$

PROBLEMA 5.6

$$x[n] \xrightarrow{DTFT} X(e^{j\omega})$$

a) $x_1[n] = x[1-n] + x[-1-n] \xrightarrow{DTFT}$ ¿ $X_1(e^{j\omega})$?

$$\begin{cases} x[n+1] \xrightarrow{DTFT} e^{j\omega} X(e^{j\omega}) \\ x[-n+1] \xrightarrow{DTFT} e^{-j\omega} \cdot X(e^{-j\omega}) \end{cases}$$

$$\begin{cases} x[n-1] \xrightarrow{DTFT} e^{-j\omega} \cdot X(e^{j\omega}) \\ x[-n-1] \xrightarrow{DTFT} e^{j\omega} \cdot X(e^{-j\omega}) \end{cases}$$

$$X_1(e^{j\omega}) = (e^{j\omega} + e^{-j\omega}) \cdot X(e^{-j\omega}) = $$
$$= \boxed{2\cos\omega \cdot X(e^{-j\omega})}$$

b) $x_2[n] = \dfrac{x^*[-n] + x[n]}{2}$

$$x^*[n] \xrightarrow{DTFT} X^*(e^{-j\omega})$$
$$x^*[-n] \longrightarrow X^*(e^{j\omega})$$

$$X_2(e^{j\omega}) = \frac{1}{2} X^*(e^{j\omega}) + \frac{1}{2} X(e^{j\omega}) = \boxed{Re\{X(e^{j\omega})\}}$$

c) $x_3[n] = (n-1)^2 \cdot x[n] = n^2 x[n] - 2n\, x[n] + x[n]$

$$n\, x[n] \xrightarrow{DTFT} j \frac{dX(e^{j\omega})}{d\omega}$$

$$n^2 x[n] = n \cdot (n\, x[n]) \xrightarrow{DTFT} - \frac{d^2 X(e^{j\omega})}{d\omega^2}$$

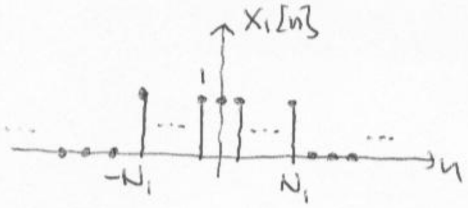
$$X_3(e^{j\omega}) = X(e^{j\omega}) - 2j \frac{dX(e^{j\omega})}{d\omega} - \frac{d^2 X(e^{j\omega})}{d\omega^2}$$

PROBLEMA 5.8

OBTENER $x[n]$ TAL QUE:

$$X(e^{j\omega}) = \frac{1}{1-e^{-j\omega}}\left(\frac{\operatorname{sen}\frac{3}{2}\omega}{\operatorname{sen}\frac{\omega}{2}}\right) + 5\pi\delta(\omega) \ , \ -\pi < \omega \leq \pi$$

- ES POSIBLE DEMOSTRAR QUE:

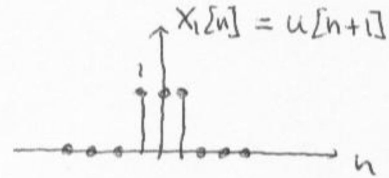


$$\xrightarrow{\text{DTFT}} X_1(e^{j\omega}) = \frac{\operatorname{sen}\left[\omega\left(N_1+\frac{1}{2}\right)\right]}{\operatorname{sen}\omega/2},$$

$$X_1(e^{j0}) = 2N_1+1$$

POR ANALOGÍA:

$$X_1(e^{j\omega}) = \frac{\operatorname{sen}\frac{3}{2}\omega}{\operatorname{sen}\omega/2} \xrightarrow{\text{DTFT}^{-1}}$$



- APLICANDO LA PROPIEDAD DE SUMA ACUMULADA:

$$x_2[n] = \sum_{k=-\infty}^{n} x_1[k] \xrightarrow{\text{DTFT}} X_2(e^{j\omega}) = \frac{X_1(e^{j\omega})}{1-e^{-j\omega}} + \pi X_1(e^{j0})\delta(\omega) \ , -\pi < \omega \leq \pi$$

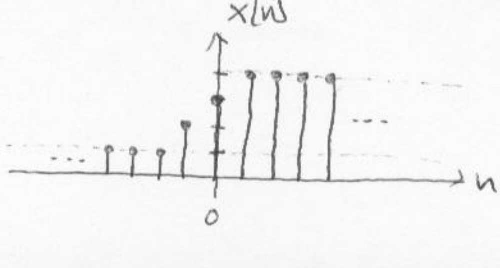
$$\Rightarrow X_2(e^{j\omega}) = \frac{1}{1-e^{-j\omega}}\cdot\left(\frac{\operatorname{sen} 3\omega/2}{\operatorname{sen}\omega/2}\right) + 3\pi\delta(\omega) \ , \ -\pi < \omega \leq \pi$$

- TENIENDO EN CUENTA QUE:

$$x_3[n] = 1 \xrightarrow{\text{DTFS}} a_0 = 1 : \xrightarrow{\text{DTFT}} X_3(e^{j\omega}) = 2\pi\delta(\omega) \ , -\pi < \omega \leq \pi$$

CONCLUÍMOS QUE:

$$x[n] = x_2[n] + x_3[n] = 1 + \sum_{k=-\infty}^{n}\left(u[k+1] - u[k-2]\right)$$



$$x[n] = \begin{cases} 1 & , n \leq 2 \\ n+3 & , -1 \leq n \leq 1 \\ 4 & , n \geq 2 \end{cases}$$

PROBLEMA 5.17

$$x[n] = (-1)^n \xrightarrow{DTFS} a_k$$
$N_0 = 2$

$$g[n] = a_n \xrightarrow{DTFS} ¿\, b_k \,?$$
$N_0 = 2$

DUALIDAD DEL DTFS:

$$x[n] \xrightarrow{DTFS} a_k$$
$$a[n] \xrightarrow{DTFS} b_k = \frac{1}{N_0} x_{-k} \Rightarrow \begin{cases} b_0 = \frac{1}{2} \\ b_1 = -\frac{1}{2} \end{cases}$$

PROBLEMA 5.19

SISTEMA LTI CAUSAL TAL QUE:

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n]$$

a) $$H(e^{j\omega}) = \frac{\sum_{k=0}^{M} b_k e^{-jk\omega}}{\sum_{k=0}^{N} a_k e^{-jk\omega}} = \frac{1}{1-\frac{1}{6}e^{-j\omega}-\frac{1}{6}e^{-2j\omega}}$$

b) ¿ $h[n]$?

$$H(e^{j\omega}) = \left.\frac{1}{1-\frac{1}{6}s-\frac{1}{6}s^2}\right|_{s=e^{-j\omega}} = \left.\frac{1}{-\frac{1}{6}(s-2)(s+3)}\right|_{s=e^{-j\omega}} =$$

$$= \frac{1}{-\frac{1}{6}(e^{-j\omega}-2)(e^{-j\omega}+3)} = \frac{1}{-\frac{1}{6}\cdot -2\left(1-\frac{1}{2}e^{-j\omega}\right)\cdot 3\left(1+\frac{1}{3}e^{-j\omega}\right)} =$$

$$= \frac{3/5}{1-\frac{1}{2}e^{-j\omega}} + \frac{2/5}{1+\frac{1}{3}e^{-j\omega}} \xrightarrow{DTFT^{-1}} h[n] = \frac{3}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{5}\left(-\frac{1}{3}\right)^n u[n] \Rightarrow$$

$$\Longrightarrow h[n] = \left[\frac{3}{5}\left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{-1}{3}\right)^n\right]\cdot u[n]$$

PROBLEMA 5.20

SISTEMA LTI CAUSAL Y ESTABLE TAL QUE:

$$\left(\frac{4}{5}\right)^n \cdot u[n] \xrightarrow{LTI} n\left(\frac{4}{5}\right)^n \cdot u[n]$$

a) ¿ $H(e^{jw})$?

$$X(e^{jw}) \xrightarrow{LTI} Y(e^{jw}) = X(e^{jw}) \cdot H(e^{jw}) \Rightarrow H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})}$$

$$x[n] = \left(\frac{4}{5}\right)^n \cdot u[n] \xrightarrow{DTFT} X(e^{jw}) = \frac{1}{1-\frac{4}{5}e^{-jw}}$$

$$y[n] = n\,x[n] \xrightarrow{DTFT} Y(e^{jw}) = j\frac{dX(e^{jw})}{dw} = j\,\frac{-1\cdot\frac{4}{5}je^{-jw}}{\left(1-\frac{4}{5}e^{-jw}\right)^2} =$$

$$= \frac{4/5\, e^{-jw}}{\left(1-\frac{4}{5}e^{-jw}\right)\left(1-\frac{4}{5}e^{-jw}\right)}$$

POR LO TANTO, $$H(e^{jw}) = \frac{4/5\, e^{-jw}}{1-4/5\, e^{-jw}}$$

b) ¿ $y[n] = f(x[n])$?

$$H(e^{jw}) = \frac{\sum_{k=0}^{M} b_k e^{-jkw}}{\sum_{k=0}^{N} a_k e^{-jkw}} \Rightarrow b_1 = \frac{4}{5},\ a_0 = 0,\ a_1 = \frac{-4}{5} \Rightarrow$$

$$\Rightarrow y[n] - \frac{4}{5}y[n-1] = \frac{4}{5}x[n-1]$$