

PROPERTIES of the DFT

The index-domain signal is $x[n]$ for $n = 0, 1, 2, \dots, N-1$; and the frequency domain values are $X[k]$ for $k = 0, 1, 2, \dots, N-1$. Outside the range $[0, N-1]$, the values of $x[n]$ and $X[k]$ are periodic.

$$\text{Analysis Equation: } X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk} \quad (1)$$

$$\text{Synthesis Equation: } x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-nk} \quad (2)$$

$$\text{Exponential: } W_N = e^{-j2\pi/N} \quad (3)$$

N -point signal: $x[n], n = 0, 1, \dots, N-1$	N -point DFT: $X[k], k = 0, 1, \dots, N-1$
$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$ (Linearity)
$x[n] = W_N^{-\ell_0 n} = e^{+j2\pi\ell_0 n/N}$	$X[k] = N \delta[(k - \ell_0) \bmod N]$
$y[n] = X[k] _{k \leftarrow n}$	$Y[k] = N \cdot x[(-k) \bmod N]$ (Duality)
$y[n] = \frac{1}{N} \cdot X[(-n) \bmod N]$	$Y[k] = x[n] _{n \leftarrow k}$ (Duality)
$x[(n - n_0) \bmod N]$	$W_N^{n_0 k} X[k]$
$W_N^{-\ell_0 n} x[n]$	$X[(k - \ell_0) \bmod N]$
$x[n] \textcircled{N} h[n] = \sum_{\ell=0}^{N-1} x[\ell] h[(n - \ell) \bmod N]$	$X[k] \cdot H[k]$
$x[n] w[n]$ (windowing)	$\frac{1}{N} \sum_{\ell=0}^{N-1} X[\ell] W[(k - \ell) \bmod N]$
$x^*[n]$	$X^*[(-k) \bmod N]$
$x^*[(-n) \bmod N]$	$X^*[k]$
$\Re\{x[n]\}$	$X_{cs}[k] = \frac{1}{2} \{X[k \bmod N] + X^*[(-k) \bmod N]\}$
$j\Im\{x[n]\}$	$X_{cas}[k] = \frac{1}{2} \{X[k \bmod N] - X^*[(-k) \bmod N]\}$
$x_{cs}[n] = \frac{1}{2} \{x[n \bmod N] + x^*[(-n) \bmod N]\}$	$\Re\{X[k]\}$ (cs = conjugate-sym)
$x_{cas}[n] = \frac{1}{2} \{x[n \bmod N] - x^*[(-n) \bmod N]\}$	$j\Im\{X[k]\}$ (cas = conj-anti-sym)
Parseval's Theorem: $N \sum_{n=0}^{N-1} x[n] ^2 = \sum_{k=0}^{N-1} X[k] ^2$	

The following properties apply when $w[n]$ is conjugate-sym:

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$\arg\{X[k]\}$ (phase) is odd

