

Unit 9: Recursion

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Objectives

- Understand and learn to use recursion

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Recursion

- A definition is recursive when it is defining something in terms of the definition itself.
- Example: The factorial function
 - “The factorial of a number is obtained multiplying the number by the factorial of the number minus 1”

$$n! = \begin{cases} n * (n-1)! & \text{If } n > 0 \\ 1 & \text{If } n = 0 \end{cases}$$

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Recursion

- A function or script is recursive when a call to itself is included in its code. The call can be direct:

```
function functionA()  
    functionA();  
end
```

or indirect:

```
function functionA()  
    functionB();  
end
```

```
function functionB()  
    functionA();  
end
```

- A recursive function includes of 2 different execution paths

- One invoking the own function (recursion)

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Recursion Example

```
numberVal = input('Introduce a number');  
rdo = factorial(numberVal);  
fprintf('The factorial of that number is %d', rdo);
```

Test.m

```
function [factVal]= factorial(n)  
% Function to compute the factorial of a number  
  
    if (n >1)  
        factVal = n* factorial (n-1);  
    else  
        factVal = 1;
```

factorial.m

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Recursion Example

```
numberVal = input('Introduce a number');  
rdo = factorial(numberVal);  
fprintf('The factorial of that number is %d', rdo);
```

3

```
function [factVal]= factorial(n)  
% Function to compute the factorial of a number  
  
    if (n >1)  
        factVal = n* factorial (n-1));  
    else  
        factVal = 1;
```

FUNCTION WORKSPACE

n	factval
3	

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Recursion Example

```
numberVal = input('Introduce a number');  
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```

3

```
function [factVal]= factorial(n)  
% Function to compute the factorial of a number  
  
    if (n >1)  
        factVal = n* factorial (n-1));  
    else  
        factVal = 1;
```

FUNCTION WORKSPACE

n	factval
3	

3

FUNCTION WORKSPACE

n	factval
2	

2

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Recursion Example

```
numberVal = input('Introduce a number');  
rdo = factorial(numberVal);  
fprintf('The factorial of that number is %d', rdo);
```

3

```
function [factVal]= factorial(n)  
% Function to compute the factorial of a number  
  
    if (n >1)  
        factVal = n* factorial (n-1));  
    else  
        factVal = 1;
```

FUNCTION WORKSPACE

n	Factval
---	---------

3

FUNCTION WORKSPACE

n	factval
---	---------

2

FUNCTION WORKSPACE

n	factval
---	---------

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Recursion Example

```
numberVal = input('Introduce a number');  
rdo = factorial(numberVal);  
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```

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```
function [factVal]= factorial(n)  
% Function to compute the factorial of a number  
  
if (n >1)  
    factVal = n* factorial (n-1));  
else  
    factVal = 1;
```

FUNCTION WORKSPACE

n	factval
3	

3	
---	--

FUNCTION WORKSPACE

n	factval
2	2

2	2
---	---

FUNCTION WORKSPACE

n	factval

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Recursion Example

```
numberVal = input('Introduce a number');  
rdo = factorial(numberVal);  
fprintf('The factorial of that number is %d', rdo);
```

3

```
function [factVal]= factorial(n)  
% Function to compute the factorial of a number  
  
if (n >1)  
    factVal = n* factorial (n-1));  
else  
    factVal = 1;
```

FUNCTION WORKSPACE

n	factorial
3	6

FUNCTION WORKSPACE

DELETED

FUNCTION WORKSPACE

DELETED

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Recursion Example

```
numberVal = input('Introduce a number');  
rdo = factorial(numberVal);  
fprintf('The factorial of that number is %d', rdo);
```

3
6

```
function [factVal]= factorial(n)  
% Function to compute the factorial of a number  
  
if (n >1)  
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Fundamental rules of recursion

- *Base cases.* You must **always** have some base case where the function can do its job without making a recursive call. **Always specify a base case otherwise indefinite recursive will occur.**
- *Always make progress.* For the cases that are to be solved recursively, the recursive call must always be to a case that makes progress toward a base case.

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Exercise

- Write a function which computes the power of a number using recursion. The function receives the number and its power. It only handles positive power numbers.

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Exercise

```
function [rdo] = myPower(num, vpow)
% Function myPpower. Returns the power of a number
%
if vpow == 0
    rdo = 1;
else
    rdo = num * myPower(num, vpow-1);
end;

end
```

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Exercise

LET'S SAY WE CALL THIS FUNCTION
LIKE THIS:

```
var = myPower(2,3)
```

```
function [rdo] = myPower(num, vpow)
% Function myPpower. Returns the power of a number
%
if vpow == 0
    rdo = 1;
else
    rdo = num * myPower(num, vpow-1);
end;

end
```

1st call - FUNCTION WORKSPACE

num	vpow	rdo
-----	------	-----



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end;

end
```

1st call - FUNCTION WORKSPACE

2nd call

num

vpow

rdo

num

vpow

rdo

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end;

end
```

1st call - FUNCTION WORKSPACE

num	vpow	rdo
-----	------	-----

2nd call

num	vpow	rdo	num	vpow	rdo
-----	------	-----	-----	------	-----

3rd call

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end
```

1st call - FUNCTION WORKSPACE

num	vpow	rdo
-----	------	-----

2nd call

num	vpow	rdo
-----	------	-----

3rd call

num	vpow	rdo
-----	------	-----

4rd call

num	vpow	rdo
-----	------	-----



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end;

end
```

1st call - FUNCTION WORKSPACE



2nd call



3rd call



4rd call



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1st call - FUNCTION WORKSPACE



2nd call



3rd call



4rd call



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end;

end
```

1st call - FUNCTION WORKSPACE

num	vpow	rdo
-----	------	-----

2nd call

num	vpow	rdo
-----	------	-----

3rd call

num	vpow	rdo
-----	------	-----

4rd call

num	vpow	rdo
-----	------	-----

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Exercise

LET'S SAY WE CALL THIS FUNCTION LIKE THIS:

```
var = myPower(2,3) ←
```

THE VALUE OF VAR WILL BE 8

```
function [rdo] = myPower(num, vpow)
% Function myPpower. Returns the power of a number
%
if vpow == 0
    rdo = 1;
else
    rdo = num * myPower(num, vpow-1);
end;
end
```

1st call - FUNCTION WORKSPACE



2nd call



3rd call



4rd call



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Exercise

- Write a function which computes the Nth number of the Fibonacci sequence, where each number is computed by summing the two preceding and starting with 0 and 1:
0, 1, 1, 2, 3, 5, 8, 13, ...

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Example

```
function [fibNum] = fibonacci(pos)
% Function fibonacci. Returns the fibonacci number
% correspondent to the position received.
switch pos
    case 1
        fibNum = 0;
    case 2
        fibNum = 1;
    otherwise
        fibNum = fibonacci(pos-1) + fibonacci(pos-2);
end;
```

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Recursion Example

```
processNumbers();  
disp('End');
```

Test.m

What does this program do?

```
function processNumbers ()  
%  
    numberVal = input('\n Introduce a number');  
    if (numberVal ~= 0)  
        processNumbers();  
        fprintf('\n %d', numberVal);  
    end;  
end
```

processNumbers.m

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Recursion Example

```
processNumbers();  
disp('End');
```

Test.m

What does this program do?

```
function processNumbers ()  
%  
    numberVal = input('\n Introduce a number');  
    if (numberVal ~= 0)  
        processNumbers();  
        fprintf('\n %d', numberVal);  
    end;  
end
```

processNumbers.m

It prints the numbers the user

introduces in reverse order until he/she introduces a 0

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Exercise

- Write a function 'reverseVector' which receives a vector and using recursion returns a vector containing the elements in reverse order.

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Exercise

```
function [ revVect ] = reverseVector( vect )
% Function reverseVectore receives a vector and returns a vector
% containing the same elements in reverse order

maxLength=length(vect);
if maxLength == 1
    revVect = vect;
else
    revVect = [vect(maxLength) reverseVector(vect(1:maxLength-1))];
end;
end
```

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Exercise

- Write a function 'sumVector' which receives a vector and using recursion returns the sum of its elements.

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Exercise

```
function [ outSum ] = sumElements ( varVect )
% Function sumElements returns the sum of the vector elements
outSum = 0;
if not (isempty(varVect))
    if length(varVect) == 1
        outSum = varVect(1);
    else
        outSum = varVect(length(varVect)) +
            sumElements(varVect(1:length(varVect)-1));
    end;
end;
```

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