Electrical Systems

Lecture 5: Electric power in AC systems II

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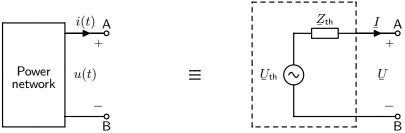
Outline

Grid Thévenin equivalent and short-circuit power

A power network is often described by its short-circuit power. Consider the power network (left) that can be represented by its Thévenin equivalent circuit (right)



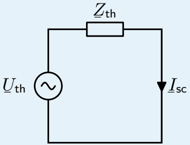
Short-circuit power of a network

Short-circuit power is the maximum power that a network (or source) is able to supply. The short-circuit power is defined as

$$S_{\text{sc}} = U_{\text{th}} I_{\text{sc}}.$$

From the Thévenin equivalent circuit, it can be obtained as

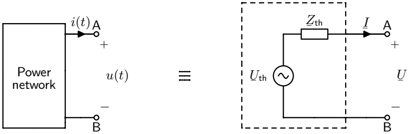
$$\underline{S}_{\text{sc}} = \underline{U}_{\text{th}} \underline{I}_{\text{sc}}^* = \frac{U_{\text{th}}^2}{\underline{Z}_{\text{th}}^*}$$



and

$$Z_{\text{th}} = \frac{U_{\text{th}}^2}{S_{\text{cc}}} \quad \left(\text{or } X_{\text{th}} = \frac{U_{\text{th}}^2}{S_{\text{cc}}} \text{ if only reactance is considered}\right)$$

Grid Thévenin equivalent and short-circuit power



Weak grids

A power grid is said to be weak if its equivalent impedance Z_{th} is high or, equivalently, if its short-circuit power is low.

In opposite...

Ideal power grid

An ideal power grid has zero equivalent impedance, $Z_{th} = 0$, and infinity short-circuit power.

Grid Thévenin equivalent and short-circuit power

Exercise 1

A load described by an impedance $\underline{Z}_L = 100 + j68\Omega$ *is connected to a power grid* $400V$*, with* $S_{sc} = 16kVA$ *and its equivalent impedance fulfils* $R_{th} = X_{th}$*.*

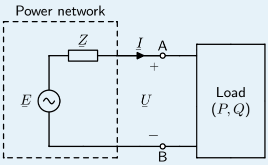
1. *Calculate the voltage at the connection point, the dissipated power in the transmission lines and the efficiency of the electrical transmission.*
2. *Repeat the calculations of the previous question assuming that* $\underline{Z}_L = 100\Omega$*.*
3. *Consider that power network is modified such that* $S_{sc} = 32kVA$ *and its equivalent impedance fulfils* $R_{th} = X_{th}$*. Calculate again the voltage at the end of the line, dissipated power and efficiency (assuming unitary power factor).*

Outline

Power network and the Thévenin equivalent circuit

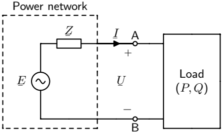
Thévenin equivalent scheme with a power defined load

Consider a load, operating at $\underline{S} = P + jQ$, connected to a power grid represented by an Thévenin equivalent circuit with $\underline{E}$ and $\underline{Z}$.



Which is the voltage across the load?

Power network and the Thévenin equivalent circuit



Writing the KVL, using that $\underline{S}^* = \underline{U}^* \underline{I}$, and after some algebra we get

The voltage across a load connected to a power grid can be calculated using...

$$U^4 + \left(2RP + 2XQ - E^2\right) U^2 + (R^2 + X^2)(P^2 + Q^2) = 0$$

or

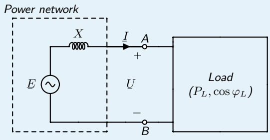
$$U^4 + \left(2\text{Re}(\underline{Z}\,\underline{S}^*) - E^2\right) U^2 + Z^2 S^2 = 0$$

Additionally, the equivalent impedance of the load, $\underline{Z}_{PQ}$, and its current and voltage (with respect to the voltage source) are, respectively,

$$\underline{Z}_{PQ} = \frac{U^2}{\underline{S}^*}, \quad \underline{I} = \frac{\underline{E}}{\underline{Z} + \underline{Z}_{PQ}}, \quad \underline{U} = \frac{\underline{Z}_{PQ}}{\underline{Z} + \underline{Z}_{PQ}} \underline{E} = \underline{Z}_{PQ} \underline{I}$$

Power network and the Thévenin equivalent circuit

Exercise 2



An electrical load is connected to a 230V power network.

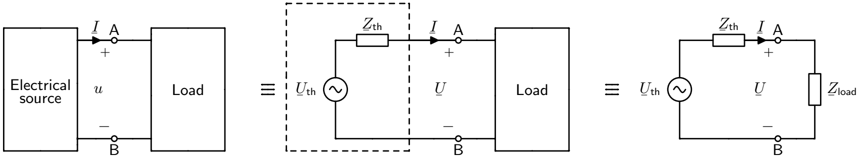
1. *Find the active power and power factor of the load if $X = 1.2\Omega$ and the power supplied by the generator is 2kW and a current measurement shows 10.87A.*
2. *Find the reactance of the power network if the measured voltage across the load is 223V and the power consumed by the load is 2kW with 0.8(i) power factor.*

Outline

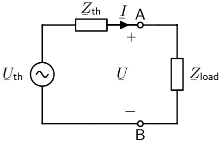
Maximum average power transfer Theorem

How we can maximize the amount of power transferred from a source to a load?

Assume we have a load connected to a power network represented by its Thévening equivalent circuit. The goal is do determine the value of the load impedance ($\underline{Z}_{\text{load}}$) that permits the maximum power delivery to $\underline{Z}_{\text{load}}$.



Maximum average power transfer Theorem



Where

$$\begin{aligned}
\underline{Z}_L &= Z_L\angle\phi_L = R_L + jX_L\\
\underline{Z}_{\text{th}} &= Z_{\text{th}}\angle\phi_{\text{th}} = R_{\text{th}} + jX_{\text{th}}\\
\underline{U}_{\text{th}} &= U_{\text{th}}\angle 0^\circ
\end{aligned}$$

The average power, or the active power, equation is given by

$$P_L = U_{L\text{rms}} I_{L\text{rms}} \cos\phi_L$$

and using

$$I_{L\text{rms}} = \frac{1}{|\underline{Z}_{\text{th}} + \underline{Z}_L|} U_{\text{th}} \qquad U_{L\text{rms}} = \frac{|\underline{Z}_L|}{|\underline{Z}_{\text{th}} + \underline{Z}_L|} U_{\text{th}} \qquad \cos\phi_L = \frac{R_L}{|\underline{Z}_L|}$$

where $|\underline{Z}_L| = \sqrt{R_L^2 + X_L^2}$ and $|\underline{Z}_{\text{th}} + \underline{Z}_L| = \sqrt{(R_L + R_{\text{th}})^2 + (X_L + X_{\text{th}})^2}$

$$P_L = \frac{R_L}{(R_L + R_{\text{th}})^2 + (X_L + X_{\text{th}})^2} U_{\text{th}}^2$$

Maximum average power transfer Theorem

For one hand, the power

$$P_L = \frac{R_L}{(R_L + R_{\text{th}})^2 + (X_L + X_{\text{th}})^2} U_{\text{th}}^2$$

has a maximum if $X_L = -X_{\text{th}}$. Then

$$P_L = \frac{R_L}{(R_L + R_{\text{th}})^2} U_{\text{th}}^2$$

the maximum power transfer for resistive case is recovered, which has a maximum for $R_L = R_{\text{th}}$.

The maximum average power transfer is given by

$$\underline{Z}_L = R_L + jX_L = R_{\text{th}} - jX_{\text{th}} = \underline{Z}_{\text{th}}^*$$

If the load impedance is purely resistive, $\underline{Z}_L = R_L$, the condition is obtained by computing $\frac{\mathrm{d}P_L}{\mathrm{d}R_L}$ to

$$P_L = \frac{R_L}{(R_L + R_{\text{th}})^2 + (X_{\text{th}})^2} U_{\text{th}}^2$$

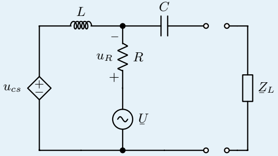
and

$$R_L = \sqrt{R_{\text{th}}^2 + X_{\text{th}}^2}$$

Maximum average power transfer Theorem

Exercise 3

From the circuit below with $R = 2\Omega$, $X_L = 4\Omega$, $X_C = 2\Omega$, $U = 4$V *and* $u_{cs}(t) = u_R(t)$,



Find the value of impedance load, $\underline{Z}_L$*, for maximum average power transfer, and the average power delivered to the load.*

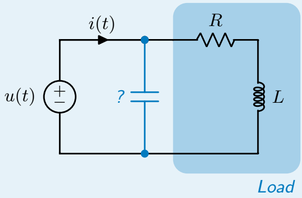
Outline

Counter-example on the reactive power definition

Exercise 4

An RL load, with values $R = 1\Omega$ and $L = 2$H, is connected to a power grid with a voltage with the form $u(t) = 10\sqrt{2}\cos(t)$V.

1. *Calculate the active, apparent, reactive powers and power factor.*
2. *Find a parallel capacitor that fully compensate the reactive power.*
3. *If $u(t) = 10\sqrt{2}\cos(5t)$V, calculate, with the designed capacitor from the previous question, the new active, apparent, reactive powers and power factor.*

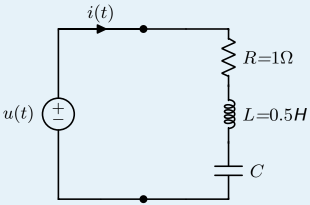


Counter-example on the reactive power definition

Exercise 5

A series RLC load is connected to a power grid with a voltage source of the form $u(t) = 100\sqrt{2}\cos(t) + 100\sqrt{2}\cos(3t)$ *V. Calculate the active, apparent, reactive powers and power factor, for both Loads A and B.*

1. *Load A: with* $C = 2/3$*F.*
2. *Load B: with* $C = 2/7$*F.*

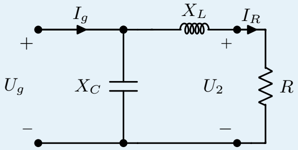


Hint: Active and apparent powers are $S = \sqrt{\sum_k U_k^2}\sqrt{\sum_k I_k^2}$ and $P = \sum_k U_k I_k \cos\phi_k$, respectively. The reactive power, from Budeanu's definition, is $Q = \sum_k U_k I_k \sin\phi_k$.

Outline

Exercises I

Exercise 6



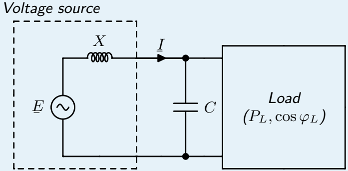
CM2: *A bulb lamp with nominal values 150V, 1500W, is connected to a 250V power grid, in series with an inductive reactance such that the voltage across the lamp is 150V. Additionally, a capacitor is connected in parallel to achieve unitary power factor. Calculate:*

1. *Reactive power of the inductor and capacitor.*
2. *Lamp and grid currents.*

Exercises II

Exercise 7

CM10: *The voltage source in the Figure on the right has a series reactance of 1Ω. The load is consuming $P_L = 10kW$ with $PF = 0.8$ (inductive) and the voltage across the load terminals is 220V. The parallel capacitor is designed such that the source current is minimal.*



Calculate:

1. *The short circuit power of the voltage source.*
2. *The value of the capacitor that ensures minimal current supplied by the voltage source.*
3. *If the load is disconnected, which is the voltage across the capacitor?*

Exercises III

Exercise 8

CM16: *A 2.4MW, 7.5kV load is connected to two voltage sources in parallel with the following properties:*

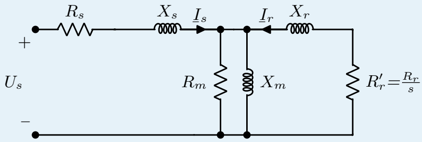
- *Voltage source 1:* $\underline{U}_{01} = 7.5\angle 0^\circ$ *kV,* $\underline{Z}_{th1} = 8 + j15\Omega$
- *Voltage source 2:* $\underline{U}_{02} = 7.0\angle 10^\circ$ *kV,* $\underline{Z}_{th2} = 5.6 + j19.2\Omega$

Find:

1. *The Thévenin equivalent circuit of the parallel voltage sources.*
2. *If the load voltage is 7.5kV, which is the total reactive power supplied by the sources?*
3. *The complex power supplied by each source.*

Exercises IV

Exercise 9



CM56: *The circuit above represents an induction motor connected to a voltage source $U_s = 220$ V, where s is the slip relating the current speed with the synchronous one. If the motor parameters are $R_s = R_s = X_s = X_r = 1\ \Omega$, $R_m = 50\ \Omega$ and $X_m = 100\ \Omega$, find:*

1. *Stator and rotor currents (I_r, I_s) when starting $(s = 1)$.*
2. *Consumed active and reactive powers when starting $(s = 1)$.*
3. *The power consumed in the resistor R_r' is proportional to the torque produced by the motor. Calculate this power when starting $(s = 1)$.*
4. *One wants to reduce the consumed current by reducing the supplied voltage U_g. Which is the required voltage if the desired current is 5 times less than the nominal one?*

Solutions I

Solution to Exercise 1

1. $U = 369.91\text{V},\ P_{\text{los}} = 66.16\text{W},\ \eta = 93.39\%$
2. $U = 372.77\text{V},\ P_{\text{los}} = 98.25\text{W},\ \eta = 93.39\%$
3. $U = 386.12\text{V},\ P_{\text{los}} = 52.70\text{W},\ \eta = 96.58\%$

Solution to Exercise 2

1. $P = 2\text{kW},\ PF = 0.827$
2. $X = 1.014\Omega$

Solution to Exercise 3

The equivalent Thévenin/Norton circuits are defined by

$$\underline{U}_{\text{th}} = 3 + j\text{V}, \underline{I}_N = 1 + j2\text{A}, \underline{Z}_{\text{th}} = 1 - j1\Omega$$

Then, the load that maximises the consumed power is $\underline{Z}_L = 1 + j1\Omega$.

Solutions II

Solution to Exercise 4

1. $P = 20\text{W}$, $S = 44.72\text{VA}$, $Q = 40\text{var}$, $PF = 0.4472$
2. $C = 0.4\text{F}$
3. $P = 0.99\text{W}$, $S = 190.1\text{VA}$, $Q = -190.1\text{var}$, $PF = 0.005$

Solution to Exercise 5

1. Load A: $P = 10\text{kW}$, $S = 14.14\text{kVA}$, $PF = 0.7071$, $Q = 0\text{var}$
2. Load B: $P = 10\text{kW}$, $S = 14.14\text{kVA}$, $PF = 0.7071$, $Q = 0\text{var}$

Solution to Exercise 6

1. $Q_L = Q_C = 2000\text{var}$
2. $I_R = 10\text{A}$, $I_g = 6\text{A}$

Solutions III

Solution to Exercise 7

1. $S_{\text{sc}} = 50.467\text{kVA}$
2. $C = 493.25\mu\text{F}$, $E = 22.465\text{V}$
3. $U_C = 265.85\text{V}$

Solution to Exercise 8

1. $U_{\text{th}} = 7.308\text{kV}$, $\underline{Z}_{\text{th}} = 3.557 + j8.526\Omega$
2. $Q = -2.057\text{Mvar}$
3. $\underline{S}_1 = 1.117 - j0.970\text{MVA}$, $\underline{S}_2 = 1.283 - j1.087\text{MVA}$

Solutions IV

Solution to Exercise 9

1. $I_r = 76.631\text{A}$, $I_s = 78.934\text{A}$
2. $P = 12.338\text{kW}$, $Q = 12.22\text{kvar}$, $PF = 0.7105$
3. $P = 5.872\text{kW}$
4. $U_s = 44\text{V}$

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