Electrical Systems

Lecture 7: Three-phase systems

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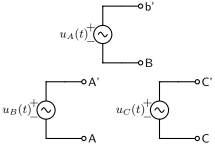
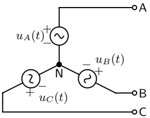
Outline

Why three-phase electrical systems?





Polyphase systems...

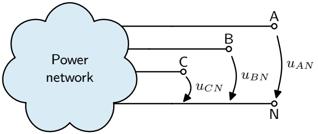
are more advantageous than single phase systems

Why three-phase electrical systems?

- In opposite to single-phase systems, in balanced three-phase system the instantaneous power is constant
- Higher efficiency with respect to the single-phase systems
- The use of cooper is reduced (around 75% of the used in single-phase systems)
- Three-phases is a trade-off between costs and efficiency of the energy transport.

Outline

Three-phase systems



A three-phase system is a set of three voltges (with the same frequency) of power network with a sequence ABC

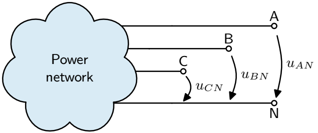
$$u_{AN}(t) = U_{AN}\sqrt{2}\cos(\omega t + \phi_A)$$
$$u_{BN}(t) = U_{BN}\sqrt{2}\cos(\omega t + \phi_B)$$
$$u_{CN}(t) = U_{CN}\sqrt{2}\cos(\omega t + \phi_C)$$

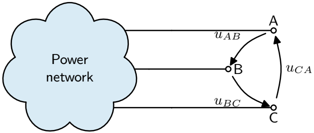
Voltages $u_{AN}(t), u_{BN}(t), u_{CN}(t)$ are referenced to a potential, usually called the neutral point (N) and can be represented using phasors as [1]

$$\underline{U}_{AN} = U_{AN}\angle\phi_A$$
$$\underline{U}_{BN} = U_{BN}\angle\phi_B$$
$$\underline{U}_{CN} = U_{CN}\angle\phi_C$$

[1] For simplicity, subindex N is sometimes omitted and $\underline{U}_A = \underline{U}_{AN}$.

Three-phase systems





Phase (or line-to-neutral) voltages

$$\underline{U}_{AN} = U_{AN}\angle\phi_A$$
$$\underline{U}_{BN} = U_{BN}\angle\phi_B$$
$$\underline{U}_{CN} = U_{CN}\angle\phi_C$$

Line (or line-to-line) voltages

$$\underline{U}_{AB} = \underline{U}_{AN} - \underline{U}_{BN}$$
$$\underline{U}_{BC} = \underline{U}_{BN} - \underline{U}_{CN}$$
$$\underline{U}_{CA} = \underline{U}_{CN} - \underline{U}_{AN}$$

Three-phase systems

A three-phase system is said to be...

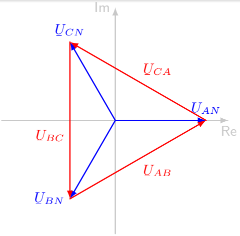
- **Symmetric** if the line voltages have the same rms value

$$U_{AB} = U_{BC} = U_{CA}$$

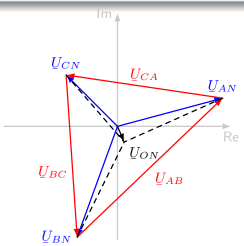
- **Balanced** if the neutral point coincides with the center of gravity of the equivalent triangle of the line voltages. Then

$$\underline{U}_{AN} + \underline{U}_{BN} + \underline{U}_{CN} = \underline{U}_{ON},$$

with $\underline{U}_{ON} = 0$.



Balanced and symmetric set of three-phase voltages



General set of three-phase voltages

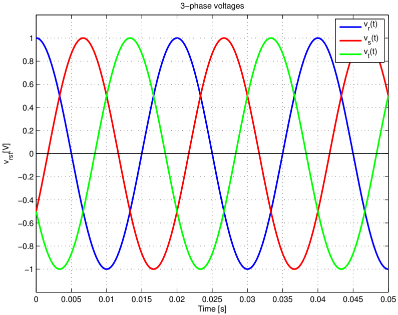
Three-phase systems

In a symmetric and balanced three-phase system the phase voltages are

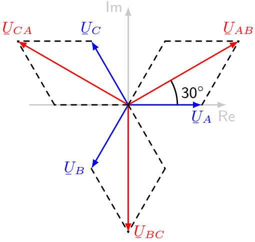
$$u_{AN}(t) = U\sqrt{2}\cos(\omega t+\phi)$$
$$u_{BN}(t) = U\sqrt{2}\cos\left(\omega t+\phi-\frac{2\pi}{3}\right)$$
$$u_{CN}(t) = U\sqrt{2}\cos\left(\omega t+\phi+\frac{2\pi}{3}\right)$$

$$\underline{U}_{AN} = U\angle\phi$$
$$\underline{U}_{BN} = U\angle(\phi-120^\circ) = a^2\underline{U}_A$$
$$\underline{U}_{CN} = U\angle(\phi+120^\circ) = a\underline{U}_A$$

where $a = 1\angle 120^\circ = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$



Three-phase systems



Equivalence between balanced line-to-line and line-to-neutral voltages

L2N voltages

$$\begin{aligned} \underline{U}_A &= U_\mathsf{P}\angle 0^\circ \\ \underline{U}_B &= U_\mathsf{P}\angle -120^\circ \\ \underline{U}_C &= U_\mathsf{P}\angle 120^\circ \end{aligned}$$

L2L voltages

$$\begin{aligned} \underline{U}_{AB} &= U_\mathsf{L}\angle 30^\circ \\ \underline{U}_{BC} &= U_\mathsf{L}\angle -90^\circ \\ \underline{U}_{CA} &= U_\mathsf{L}\angle -210^\circ \end{aligned}$$

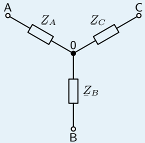
$$U_\mathsf{L} = \sqrt{3} U_\mathsf{P}$$

Outline

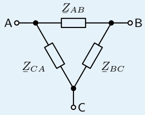
Three-phase loads

Y-Δ loads

Y (Wye) connected load*



Δ (Delta) connected load



* Node 0 in Y-loads can be connected (or disconnected) to the neutral of the generator.

A load is said to be balanced if...

- $\underline{Z}_Y = \underline{Z}_A = \underline{Z}_B = \underline{Z}_C$,

or

- $\underline{Z}_\Delta = \underline{Z}_{AB} = \underline{Z}_{BC} = \underline{Z}_{CA}$,

Three-phase loads

Kennelly's Theorem (or Y-Δ transformation)

$$Z_{AB} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_C} \qquad Z_A = \frac{Z_{AB} Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

$$Z_{BC} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_A} \qquad Z_B = \frac{Z_{BC} Z_{AB}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

$$Z_{CA} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_B} \qquad Z_C = \frac{Z_{CA} Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

Note that, if...

$$Z_A = Z_B = Z_C = Z_Y \iff Z_{AB} = Z_{BC} = Z_{CA} = Z_\Delta$$

Then

$$Z_\Delta = 3 Z_Y$$

Three-phase loads

Exercise 1

A balanced delta-connected load contains a 10Ω *resistor in series with a* $20mH$ *inductor in each phase. The voltage source is a balanced* abc*-sequence three-phase* $50Hz$ *with a line voltage of* $100V$*.*

1. *Find all the line currents.*
2. *If the impedance between phases* a *and* b *is removed, find again all the line currents.*

Three-phase loads

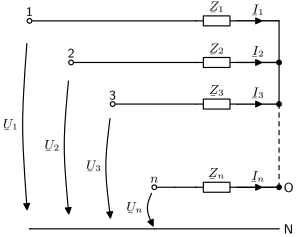
Solution: Matlab code

```
clear all; close all; clc

U=100;
Z=10+1i*100*pi*20e-3;

a=cos(2*pi/3)+1i*sin(2*pi/3);
Uan=U/sqrt(3);
Ubn=U/sqrt(3)*a^2;
Ucn=U/sqrt(3)*a;

disp('--- Question 1 ---')
Iab=(Uan-Ubn)/Z;
disp(['Iab=' num2str(abs(Iab)) ',' num2str(180/pi*angle(Iab)) 'A'])
Ibc=(Ubn-Ucn)/Z;
Ica=(Ucn-Uan)/Z;
disp(['Ica=' num2str(abs(Ica)) ',' num2str(180/pi*angle(Ica)) 'A'])
Ia=Iab-Ica;
disp(['Ia=' num2str(abs(Ia)) ',' num2str(180/pi*angle(Ia)) 'A'])
Ib=Ibc-Iab;
disp(['Ib=' num2str(abs(Ib)) ',' num2str(180/pi*angle(Ib)) 'A'])
Ic=Ica-Ibc;
disp(['Ic=' num2str(abs(Ic)) ',' num2str(180/pi*angle(Ic)) 'A'])

disp('--- Question 2 ---')
Ia=-Ica;
disp(['Ia=' num2str(abs(Ia)) ',' num2str(180/pi*angle(Ia)) 'A'])
Ib=Ibc;
disp(['Ib=' num2str(abs(Ib)) ',' num2str(180/pi*angle(Ib)) 'A'])
Ic=Ica-Ibc;
disp(['Ic=' num2str(abs(Ic)) ',' num2str(180/pi*angle(Ic)) 'A'])
```

Outline

Millman's Theorem

Consider a load with n different impedances connected to a floating point, O, with respect to the neutral of the polyphase system, N.



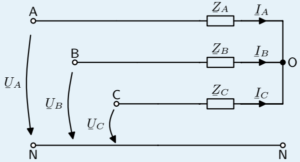
The voltage $\underline{U}_{ON}$ can be obtained, from the phase voltages with the result of the Millman's Theorem.

Millman's Theorem

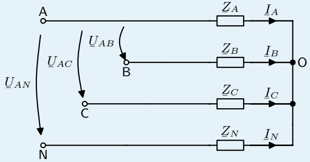
$$\underline{U}_{ON} = \frac{\sum_{k=1}^{n} \underline{U}_{kN}\underline{Y}_k}{\sum_{k=1}^{n} \underline{Y}_k}$$

Millman's Theorem

Applications of Millman's Theorem to three phase loads



$$\underline{U}_{ON} = \frac{\underline{U}_{AN}\underline{Y}_A + \underline{U}_{BN}\underline{Y}_B + \underline{U}_{CN}\underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C}$$



$$\underline{U}_{AO} = \frac{\underline{U}_{AB}\underline{Y}_B + \underline{U}_{AC}\underline{Y}_C + \underline{U}_{AN}\underline{Y}_N}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C + \underline{Y}_N}$$

or, in the particular case of $\underline{Y}_N = 0$ ($\underline{Z}_N = \infty$),

$$\underline{U}_{AO} = \frac{\underline{U}_{AB}\underline{Y}_B + \underline{U}_{AC}\underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C}$$

Millman's Theorem

Exercise 2

A balanced wye-connected load contains a 10Ω resistor in series with a $20mH$ inductor in each phase. The voltage source is a balanced abc-sequence three-phase $50Hz$ with a line voltage of $100V$.

1. *Find all the line currents.*
2. *If in phase a, the resistor is short-circuited, find again all the line currents and the voltage U_{on}.*

Millman's Theorem

Solution: Matlab code

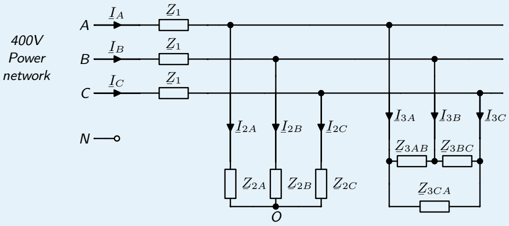
```
clear all; close all; clc

U=100;
Z=10+1i*100*pi*20e-3;

a=exp(1i*2*pi/3);
Uan=U/sqrt(3);
Ubn=U/sqrt(3)*a^2;
Ucn=U/sqrt(3)*a;

disp('--- Question 1 ---')
Ia=Uan/Z;
disp(['Ia=' num2str(abs(Ia)) ',' num2str(180/pi*angle(Ia)) 'A'])
Ib=Ubn/Z;
disp(['Ib=' num2str(abs(Ib)) ',' num2str(180/pi*angle(Ib)) 'A'])
Ic=Ucn/Z;
disp(['Ic=' num2str(abs(Ic)) ',' num2str(180/pi*angle(Ic)) 'A'])

disp('--- Question 2 ---')
Za=1i*100*pi*20e-3;
Zb=Z;Zc=Z;

Uon=(Uan/Za+Ubn/Zb+Ucn/Zc)/(1/Za+1/Zb+1/Zc);
disp(['Uon=' num2str(abs(Uon)) ',' num2str(180/pi*angle(Uon)) 'V'])

Ia=(Uan-Uon)/Za;
disp(['Ia=' num2str(abs(Ia)) ',' num2str(180/pi*angle(Ia)) 'A'])
Ib=(Ubn-Uon)/Zb;
disp(['Ib=' num2str(abs(Ib)) ',' num2str(180/pi*angle(Ib)) 'A'])
Ic=(Ucn-Uon)/Zc;
disp(['Ic=' num2str(abs(Ic)) ',' num2str(180/pi*angle(Ic)) 'A'])
```

Outline

Exercises I

Exercise 3

Given the three-phase circuit below with impedances $\underline{Z}_1 = 0.5 + j\Omega$, $\underline{Z}_{2A} = \underline{Z}_{2B} = \underline{Z}_{2C} = 15 + j8\Omega$, *and* $\underline{Z}_{3AB} = \underline{Z}_{3BC} = \underline{Z}_{3CA} = 30 - j12\Omega$



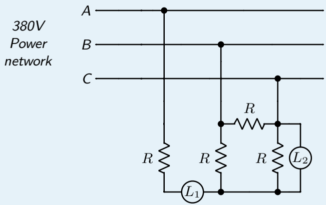
Find:

1. *The line currents* (I_A, I_B, I_C) *and the voltage at* U_{ON}.
2. *The same voltage and the line currents, if* $\underline{Z}_1 = 0$ *and* $\underline{Z}_{2A} = j8\Omega$.

Exercises II

Exercise 4

CT25: *In the circuit in the figure, the power grid is symmetric and balanced,* $R = 5\Omega$*, and* L_1*,* L_2 *are an ammeter and a voltmeter, respectively.*



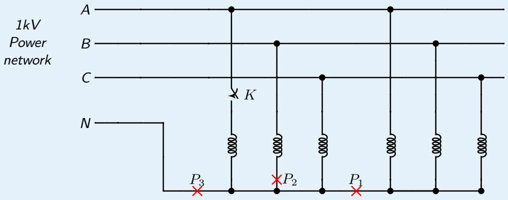
Find:

1. *The* L_1, L_2 *measurements.*
2. *If, erroneously, we connect two ammeters, which are the new measures?*
3. *If, erroneously, we connect two voltmeters, which are the new measures?*

Exercises III

Exercise 5

CT36: *In the circuit of the figure, the power grid is symmetric and balanced, all the reactances are $X = 100\Omega$. A failure occurs and switch K is disconnected. Find:*



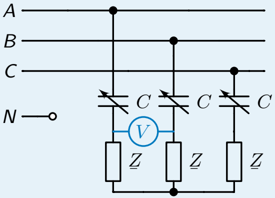
The measure of:

1. *a voltmeter placed in point P_1.*
2. *an ammeter placed in point P_1.*
3. *a voltmeter placed in point P_2.*
4. *an ammeter placed in point P_2.*
5. *a voltmeter placed in point P_3.*
6. *an ammeter placed in point P_3.*

Exercises IV

Exercise 6

The circuit in the Figure below shows a direct sequence three-phase system of 400 *V.*

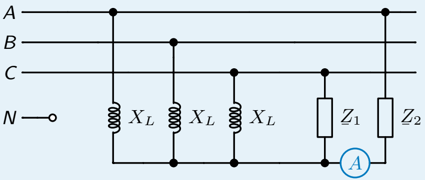


It the variable capacitors are equal in each phase and the three impedances have value $\underline{Z} = 8 + j6\ \Omega$*, find the maximum voltage measured by the voltmeter when changing the capacitance values.*

Exercises V

Exercise 7

Find the measure of the ammeter of the three-phase system in the Figure below.



Solutions I

Solution to Exercise 1

(Using $\underline{U}_{an}$ as the reference voltage)

1. $\underline{I}_a = 14.66\angle -32.14°$A, $\underline{I}_b = 14.66\angle -152.14°$A, $\underline{I}_c = 14.66\angle 87.85°$A ($\underline{I}_{ab} = 8.47\angle -2.14°$A)
2. $\underline{I}_a = 8.47\angle -62.14°$A, $\underline{I}_b = 8.47\angle -122.14°$A, $\underline{I}_c = 14.67\angle 87.85°$A

Solution to Exercise 2

(Using $\underline{U}_{an}$ as the reference voltage)

1. $\underline{I}_a = 4.89\angle -32.14°$A, $\underline{I}_b = 4.89\angle -152.14°$A, $\underline{I}_c = 4.89\angle 87.85°$A
2. $\underline{I}_a = 8.11\angle -62.05°$A, $\underline{I}_b = 4.15\angle 180°$A, $\underline{I}_c = 7.18\angle 87.2°$A, $\underline{U}_{on} = 27.06\angle -62.05°$V

Solutions II

Solution to Exercise 3

1. $I_A = 29.8\text{A},\ I_B = 29.8\text{A},\ I_C = 29.8\text{A},\ U_{ON} = 0\text{V}$
2. $I_A = 35.28\text{A},\ I_B = 26.04\text{A},\ I_C = 38.39\text{A},\ U_{ON} = 122.4\text{V}$

Solution to Exercise 4

1. $L_1 = 43.88\text{A},\ L_2 = 219.39\text{V}$
2. $L_1 = 76\text{A},\ L_2 = 131.63\text{A}$
3. $L_1 = 329.09\text{V},\ L_2 = 190\text{V}$

Solutions III

Solution to Exercise 5

1. $U_1 = 0\text{V}$
2. $I_1 = 0\text{A}$
3. $U_2 = 577.35\text{V}$
4. $I_2 = 5.77\text{A}$
5. $U_3 = 115.47\text{V}$
6. $I_3 = 5.77\text{A}$

Solution to Exercise 6

$$V = 500 \text{ V}$$

Solutions IV

Solution to Exercise 7

$$A = \frac{1}{\underline{Z}_2}\left(\frac{\underline{U}_{AB}\frac{1}{jX_L} + \underline{U}_{AC}\left(\frac{1}{jX_L} + \frac{1}{\underline{Z}_1}\right)}{\frac{3}{jX_L} + \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2}}\right)$$

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