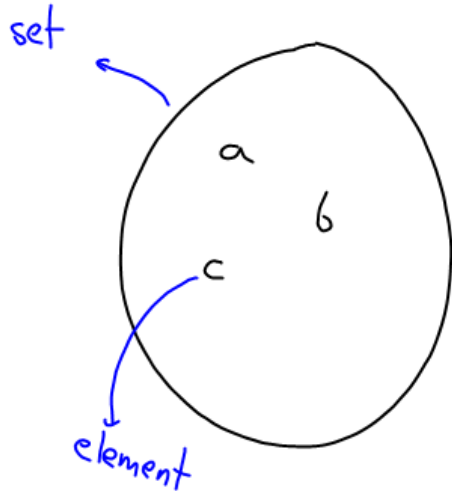


Sets



Relation \equiv law or simple relation between two elements of a set.



Equivalence relations \equiv A relation that follows:

1. Reflexion $a R a \quad \forall a \in E$
2. Symmetry $a R b \longrightarrow b R a \quad \forall a, b \in E$
3. Transitivity $a R b \wedge b R c \longrightarrow a R c \quad \forall a, b, c \in E$

Example:



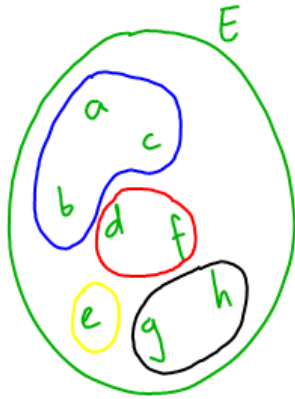
Given:
 $a R b$

If we know R is an ER ,
what does R do with E .

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70

Cartagena99



R forms a Partition in E : E/R

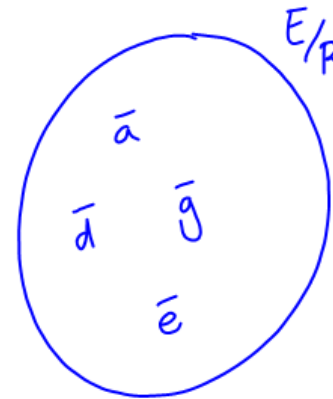
Equivalence class : $\bar{x} = \{y \in E / x R y\}$

$$\bar{a} = \{a, b, c\} = \bar{b} = \bar{c}$$

$$\bar{d} = \{d, f\} = \bar{f}$$

$$\bar{g} = \{g, h\} = \bar{h}$$

$$\bar{e} = \{e\}$$



Cartagena99

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70

GROUPS

A group is a set with an ILC (internal law of composition)

$$(G, *) \quad * \text{ is an ILC} \iff \begin{cases} \text{Internal } \forall a, b \in G \ a * b \in G \\ \exists \text{ Neutral element } \exists ! e \in G / \forall a \in G \ a * e = e * a = a \\ \text{Symmetrical } \forall a \in G \ \exists a' \in G / a * a' = a' * a = e \end{cases}$$

\downarrow set \downarrow ILC

Example: $(\mathbb{Z}, +)$

Example:

$$G = \{1, -1, i, -i\}$$

\cdot = Natural multiplication of \mathbb{C}

\cdot	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

INTERNAL ✓
Neutral: 1 ✓

Autosymmet. ✓

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70

Cartagena99

Autosymmetrical \equiv an element that acts as its own symmet. $\forall a \in G \quad a = a' \quad a * a = e$

Abelian Group \equiv group where $\forall a, b \in G \quad a * b = b * a$

Finite groups Groups that have a finite number of elements called the group's ORDINAL (or order).

$$(G = \{1, -1, i, -i\}, \cdot) \quad o(G) = 4$$

Order of an element Number of times we must write an element operated with itself to obtain the neutral element.

$$(G, \cdot) \quad o(i) = 4 \quad \text{because } i \cdot i \cdot i \cdot i = 1$$

Lagrange's Law . Inside a finite group the order of any element is a divisor of the order of the group.

\rightarrow The Neutral element ALWAYS has order 1

Cartagena99

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70

$$o(i) = 4$$

Subgroups Given a group $(G, *)$, we can say that $S \subseteq G$ is a
 subgroup of $G \iff (S, *)$ is a Group -
→ contained or equal to

Also:

$$S \text{ is subgroup of } G \iff \forall a, b \in S \quad a * b' \in S$$

Lagrange's law for subgroups: A subgroup's order is always divisor of the groups order.

Example: (G, \cdot)
 $G = \{1, -1, i, -i\}$

Orders of subgroups can only be 1, 2 and 4

$$o(S_1) = 1 \rightarrow S_1 = \{1\} \quad \begin{array}{c|c} & 1 \\ \hline 1 & 1 \end{array}$$

$$o(S_2) = 2 \rightarrow S_2 = \{1, -1\} \quad \begin{array}{c|cc} & 1 & -1 \\ \hline 1 & 1 & -1 \\ -1 & -1 & 1 \end{array}$$

$$o(S_3) = 4 \rightarrow S_3 = \{1, -1, i, -i\}$$

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
 LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
 CALL OR WHATSAPP: 689 45 44 70

Cartagena99

Cyclical groups If a finite group has an element of the same order as the group itself, that group is called cyclical, and that element is called a generator.

$$(G, \cdot) \quad \left. \begin{array}{l} o(G) = 4 \\ o(i) = 4 \end{array} \right\} \rightarrow G \text{ is cyclical} \rightarrow i \text{ is a generator of } G$$

$$\begin{array}{l} i \\ \hline i \cdot i = -1 \\ i \cdot i \cdot i = -i \\ i \cdot i \cdot i \cdot i = 1 \\ \hline i \cdot i \cdot i \cdot i \cdot i = i \\ i \cdot i \cdot i \cdot i \cdot i \cdot i = -1 \\ i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i = -i \\ i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i = 1 \\ \hline i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i = i \\ \hline \vdots \end{array} \quad \begin{array}{l} \text{1st} \\ \text{2nd} \end{array}$$

Cartagena99

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70

Normal Subgroups When a subgroup $H \subset G$ follows these rules, it is called a Normal Subgroup:

- $$\left\{ \begin{array}{l} 1. \forall a \in G \quad a * H \subseteq H \\ \text{OR} \\ 2. \forall a \in G \quad H * a \subseteq H \\ \text{OR} \\ 3. \forall a \in G \quad a * H = H * a \end{array} \right.$$

If G is Abelian then every subgroup in G is NORMAL.

Cartagena99

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70