Airfoils

Kutta condition (I)

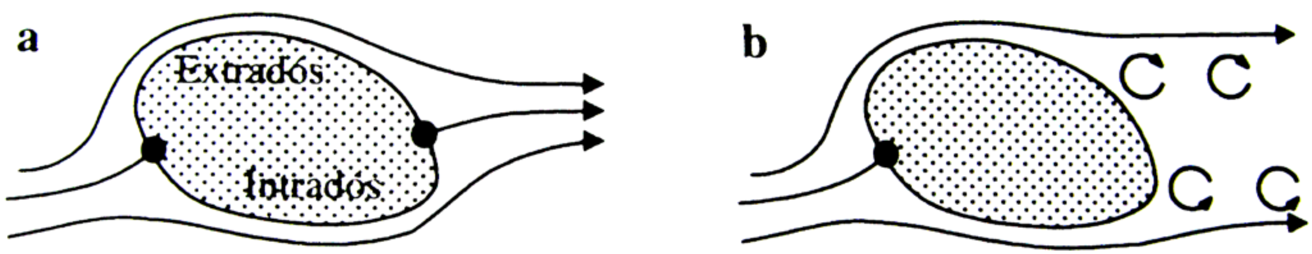
- We remember that: $\left.\begin{array}{l}\rho = cte \\ \mu = 0 \\ \vec{\mathbf{f}}_m = 0\end{array}\right\} \quad \nabla \times \vec{\mathbf{V}} = 0$

  The circulation is constant in any point in the flow field.

- How can we obtain lift if we need circulation and the circulation in the upstream farfield is zero?

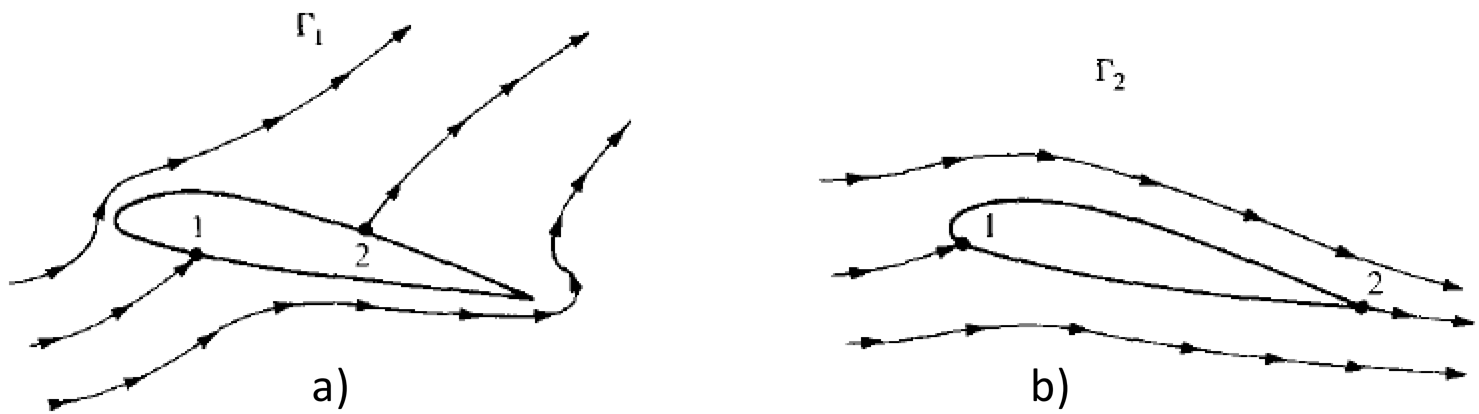
Any body immersed in a free stream that can be treated as a potential fluid, shows two separatix streamlines that touch the body in the stagnation points. Those stagnation points define the upper and lower surface of the profile.

But in reality, the viscosity modifies the velocity profile and the pressure, thus the flow detaches before the second stagnation point.



Kutta condition (II)

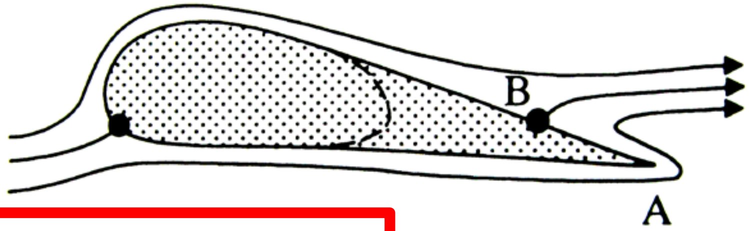
- We have seen the cylinder example where many solutions are available depending on the value of the circulation Γ
- Similar approach can be applied in a generic profile where many potential solutions fit the shape of the profile.



- The only difference between these two cases a) and b) is their circulation Γ

Kutta condition (III)

- We know that the flow moves towards the lower pressure areas , and when it has to overcome an adverse pressure gradient, it has to pay with its kinetic energy.
- If we had a flow as shown in the figure, the flow should bend in the corner A and in theory, the velocity in this point should be infinite, and it would have to strongly decelerate between A and B.



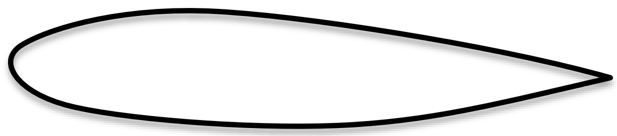
Kutta condition: the circulation around the profile is the one that

- either sets a stagnation point in the trailing edge
- or removes the second stagnation point

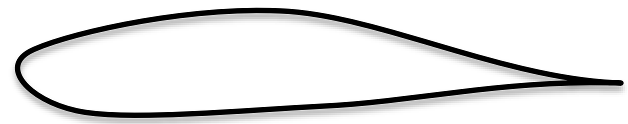
- We cannot have infinite velocity, so we cannot have infinite pressure gradient, then, the upper and lower surface pressure in the trailing edge must be equal.

Kutta condition (IV)

- The Kutta condition shows two different behaviours:



a)



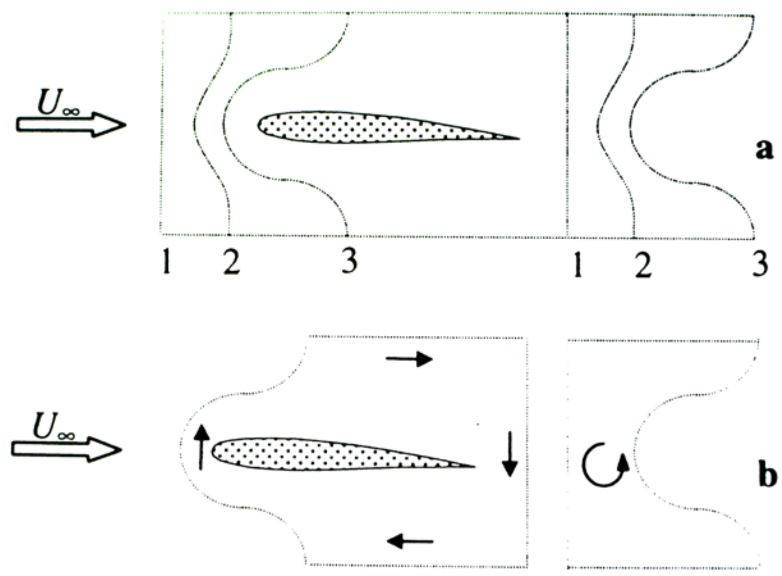
b)

a) The trailing edge has two different tangents, thus the only option in order to avoid having different velocities in the same point is that those velocities should be zero (stagnation point)

b) The upper and lower surfaces have the same slope in the trailing edge and we can have a non-zero velocity that is identical in the upper and lower surface.

Profile circulation

- We know that before the movement starts we do not have any circulation; and according to the potential model, we cannot create circulation with the time evolution.
- We can explain the generation of the circulation because we start from a static situation and we consider a fluid volume around the profile. But A closed curve will remain closed and thus we can consider the circulation in this closed curve will remain constant and equal to zero. However, we can split the curve in two, and we can justify the existence of circulation around the profile because we have left an opposite circulation in the downwash.



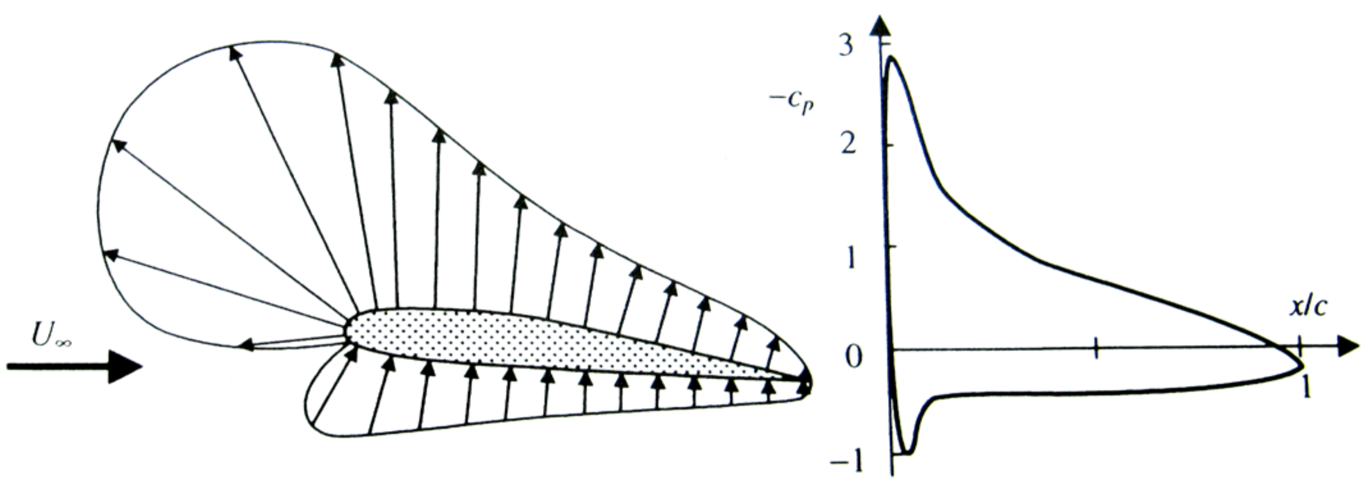
Coefficients

- The pressure coefficient is:

$$c_p\left[x, z_p(x)\right] = \frac{p\left[x, z_p(x)\right] - p_\infty}{\frac{1}{2}\rho U_\infty^{\ 2}}$$

- If we consider the component of the pressure force that is perpendicular to the incoming flow direction we obtain the same $c_p(x)$.
- The lift distribution is:

$$c_l(x) = c_{p_{\text{lower surface}}}(x) - c_{p_{\text{upper surface}}}(x)$$



Coefficients

- The lift and lift coefficient are:

$$l = \frac{1}{2}\rho U_{\infty}{}^{2} c\, c_l$$

$$c_l = \frac{1}{c}\int_{x_{le}}^{x_{te}} c_l(x)\,dx$$

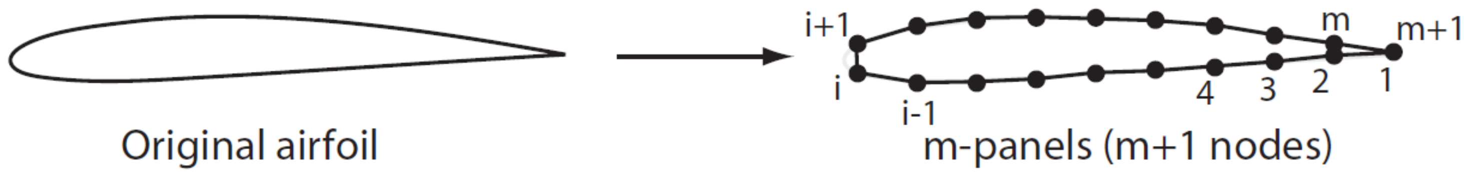
- **Aerodynamic center of a profile**: is the chord-wise length about which the pitching moment is independent of the lift coefficient and the angle of attack.
  - It is located in:
    - Subsonic $x = \frac{c}{4}$
    - Supersonic $x \approx \frac{c}{2}$

$$m_{ac} = \frac{1}{2}\rho U_{\infty}{}^{2} c^{2}\, c_{m_{ac}}$$

$$c_{m_{ac}} = \frac{-1}{c^{2}}\int_{x_{le}=-c/2}^{x_{te}=c/2} c_l(x)\left(x + \frac{c}{4}\right)dx$$

Vortex panel method

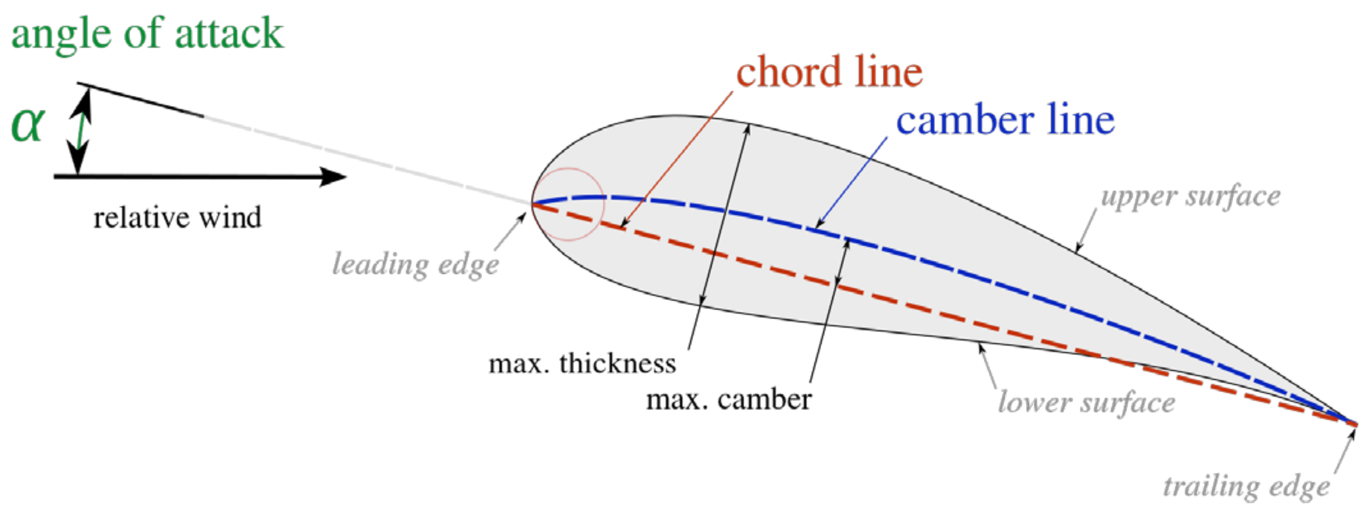
- We replace the profile by a set of panels with distributed singularities (vortex)



- The strength of the singularities is calculated so that the velocity in a certain point of the panel (usually the center of the panel) is tangent to the panel.
- There are still more variables than equations and we need to impose the Kutta condition.
- Finally our problem can be solved as a linear system.

Thin airfoil theory

- The leading edge is the point at the front of the airfoil that has maximum curvature.
- The trailing edge is defined similarly as the point of maximum curvature at the rear of the airfoil.
- Chord line is a straight line going from the leading edge (point of minimum radius) to the trailing edge.
- Mean camber line is constituted by the midpoints of all airfoil cross-section segments perpendicular to the chord.
- The thickness of an airfoil is measured perpendicular to the chord line.



Thin airfoil theory

- The Laplace equation that describes the potential of velocities is:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$$

- And the boundary conditions we have to apply are:
  - Velocity tangent to the surface $z = z_p(x)$

$$\left.\frac{\Phi_z(x,z)}{\Phi_x(x,z)}\right|_{z=z_p} = \frac{dz_p}{dx}$$

  - Kutta condition in the trailing edge
  - Non-perturbed free flow in the infinite upwash:

$$\Phi(x,z) \to U_\infty x \quad \text{cuando} \quad x^2 + z^2 \to \infty$$

- The pressure coefficient is:

$$c_p(x,z) = 1 - \frac{\Phi_x{}^2(x,z) + \Phi_z{}^2(x,z)}{U_\infty{}^2}$$

Thin airfoil theory: hypothesis

- Incompressible
- Inviscid
- Irrotational
- Small angle of attack
- Small airfoil thickness
- Small camber

$$z = \varepsilon Z_p(x) \quad \text{where} \quad a \leq x \leq b \quad \text{and} \quad \varepsilon << 1$$

Thin airfoil theory: linearization

- Considering small thickness, curvature and angle of attack, we can suppose that perturbations of the velocity are small too.

$$z = \varepsilon Z_p(x) \qquad \text{where} \quad a \le x \le b \quad \text{and} \quad \varepsilon << 1$$

$$\Phi(x,z) = U_\infty x + \varphi(x,z) = U_\infty x + \delta(\varepsilon)\underbrace{\varphi'(x,z)}_{\mathrm{O}(1)}$$

- After linearizing this problem and neglecting terms of order ε, we conclude that the boundary conditions are applied on $z=0$; then we can apply superposition principle.
- Finally, we obtain the following:

differential equation: $\dfrac{\partial^2 \varphi}{\partial x^2} + \dfrac{\partial^2 \varphi}{\partial z^2} = 0$

and boundary conditions: $\left.\dfrac{\varphi_z(x,z)}{U_\infty}\right|_{z=0^\pm} = \dfrac{dz_p}{dx} \quad \text{in } z = 0^\pm,\ a \le x \le b$

$$\varphi_x(x,0^+) = \varphi_x(x,0^-) \quad \text{in } x = b,\ z = 0$$

$$\varphi(x,z) \to 0 \quad \text{in } x^2 + z^2 \to \infty$$

- The pressure coefficient results: $c_p(x,0^\pm) = -2\dfrac{\varphi_x(x,0^\pm)}{U_\infty} \quad \text{in } a \le x \le b, \quad z = 0^\pm$

Thin airfoil theory: linearization

We have seen that the thin airfoil theory allows superposition of the potential of velocities; thus we can linearly decompose any problem into two solutions.

- **Anti-symmetric problem:** **ANGLE OF ATTACK + CURVATURE**

- **Symmetric problem:** **THICKNESS**

PROBLEM	w	φ	u
Curvature	S	A	A
Angle of attack	S	A	A
Thickness	A	S	S

Note

- **Derivation with respect to z changes the symmetry**
- **Derivation with respect to x does not change the symmetry**
- **Solution φ has the same symmetry as the geometric boundary condition**

Thin airfoil theory: linearization

- We can analyze the velocity in the upper and lower curve.

$$u(x,z)=U_\infty+\underbrace{\tilde{u}(x,z)}_{\varphi_x(x,z)} \qquad c_p(x,z)=-2\frac{\tilde{u}(x,z)}{U_\infty}$$

- The distributed circulation is

$$\gamma(x)=\underbrace{\tilde{u}(x,0^+)}_{\tilde{u}_{upper}}-\underbrace{\tilde{u}(x,0^-)}_{\tilde{u}_{lower}}$$

and as a result

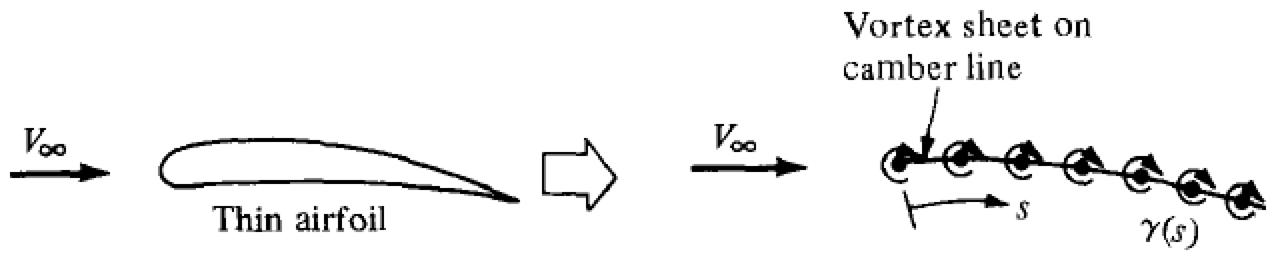
$$c_l(x)=2\frac{\gamma(x)}{U_\infty}=4\frac{\tilde{u}(x)}{U_\infty}$$

Thin airfoil theory: symmetric problem (φ)

- In a symmetric problem (no curvature and no angle of attack) we expect the same pressure in the upper an lower surface.
  - Lift is zero
  - Drag is zero (potential flow)

Thin airfoil theory: anti-symmetric problem (φ)

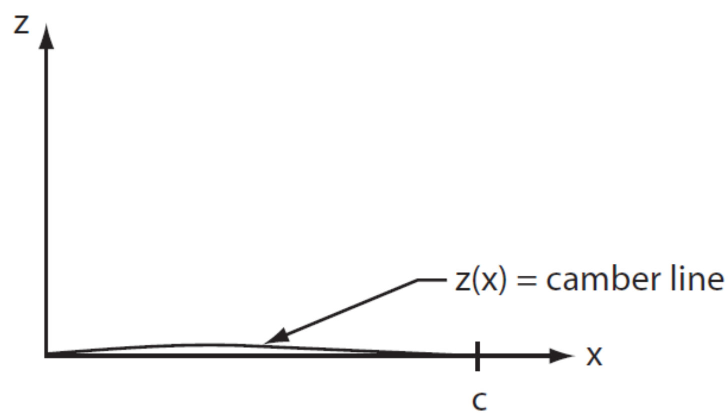
- A profile whose thickness is much smaller than its chord can be considered thin, and if it is observed from the distance, the usual panel method would be seen as two lines of distributed vortex, very close one to the other. From the distance, they are so close, that they can be considered as an only line.
- In this approximation the profile has zero thickness and the camber line will be one streamline in our problem.

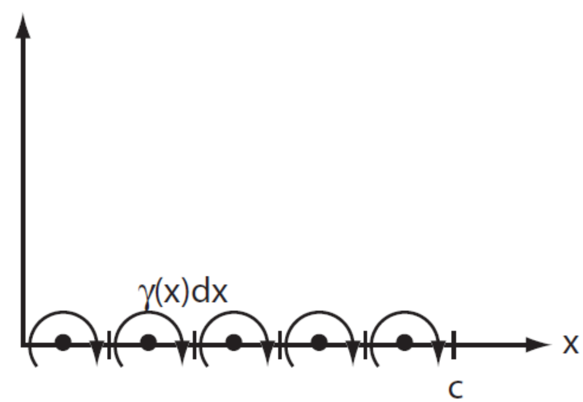


Thin airfoil theory: anti-symmetric problem (φ)

- We need to impose the camber line to be a streamline.
- Following the similar reasoning that led us from the real airfoil with upper and lower surface to the camber line to be an streamline, there is no much difference if we consider the vortex distribution to be placed in the chord line and not in the camber line.

$$U_\infty\left(\alpha-\frac{dz_p}{dx}\right)-\underbrace{\int_0^c\frac{\gamma(\xi)d\xi}{2\pi(x-\xi)}}_{\text{vertical velocity induced by the distributed vortex}}=0 \qquad \alpha\approx\sin\alpha\approx\operatorname{tg}\alpha$$





Thin airfoil theory: anti-symmetric problem (φ)

- The commonly used change of variable is

$$\xi = \frac{c}{2}\left(1-\cos\theta\right) \quad , \qquad \theta \in \left[0,\pi\right]$$

- The problem will be decomposed into

ANGLE OF ATTACK + CURVATURE

Thin airfoil theory: ANGLE OF ATTACK (flat plate)

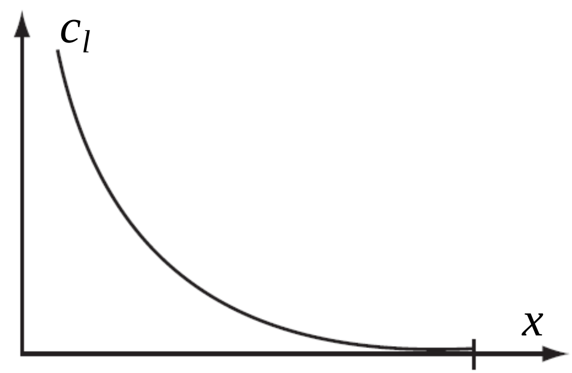
- The vortex strength is:

$$\gamma(\theta) = 2\alpha U_\infty \frac{1+\cos\theta}{\sin\theta} \qquad \Rightarrow \qquad \gamma(x) = 2\alpha U_\infty \sqrt{\frac{1-\frac{x}{c}}{\frac{x}{c}}}$$

- Thus, lift coefficient is $c_l(x) = c_{p_{lower}} - c_{p_{upper}} = 2\frac{\gamma(x)}{U_\infty} = -4\frac{u(x)}{U_\infty}$

  and as a result

$$\boxed{c_l(x) = 4\alpha \sqrt{\frac{1-\frac{x}{c}}{\frac{x}{c}}}}$$



- Conclusions
  - ✓ $c_l=0$ at trailing edge (Kutta condition requires $p_{upper}=p_{lower}$)
  - ✓ $c_l=\infty$ at leading edge ("suction peak")
    - o Suction peak is not desirable because it can result in:
      1. Leading edge separation
      2. Very low pressure at leading edge which must rise towards trailing edge $\Rightarrow$ adverse pressure gradients $\Rightarrow$ boundary layer separation

Thin airfoil theory: CURVATURE + ANGLE OF ATTACK

- We can use a Fourier series for the vortex strength distribution:

$$\gamma(\theta) = 2U_\infty \left[\underbrace{A_0 \frac{1+\cos\theta}{\sin\theta}}_{\substack{\text{flat plate with} \\ \text{angle of attack}}} + \underbrace{\sum_{n=1}^{\infty} A_n \sin(n\theta)}_{\text{cambered contribution}} \right]$$

where

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx} d\theta_0$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx} \cos(n\theta_0) d\theta_0$$

and finally $c_l = 2\pi(\alpha - \alpha_{L=0})$

$$c_{m_{c/4}} = c_{m_{ac}} = \frac{\pi}{4}(A_2 - A_1)$$

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^\pi \frac{dz}{dx}(\cos\theta_0 - 1) d\theta_0 = \alpha - A_0 - \frac{1}{2}A_1$$

- Conclusions
  - ✓ In thin airfoil theory, the aerodynamic center is always at the quarter-chord $c/4$, regardless of the airfoil shape or angle of attack
  - ✓ **Lift slope** $\boxed{\dfrac{dc_l}{d\alpha} = 2\pi}$
  - ✓ **Ideal angle of attack:** is the angle of attack that makes disappear the leading edge singularity ($A_0=0$)

Exercise

1. Consider a thin symmetric airfoil at 1.5° angle of attack. Calculate lift and moment in the leading edge.

Exercise

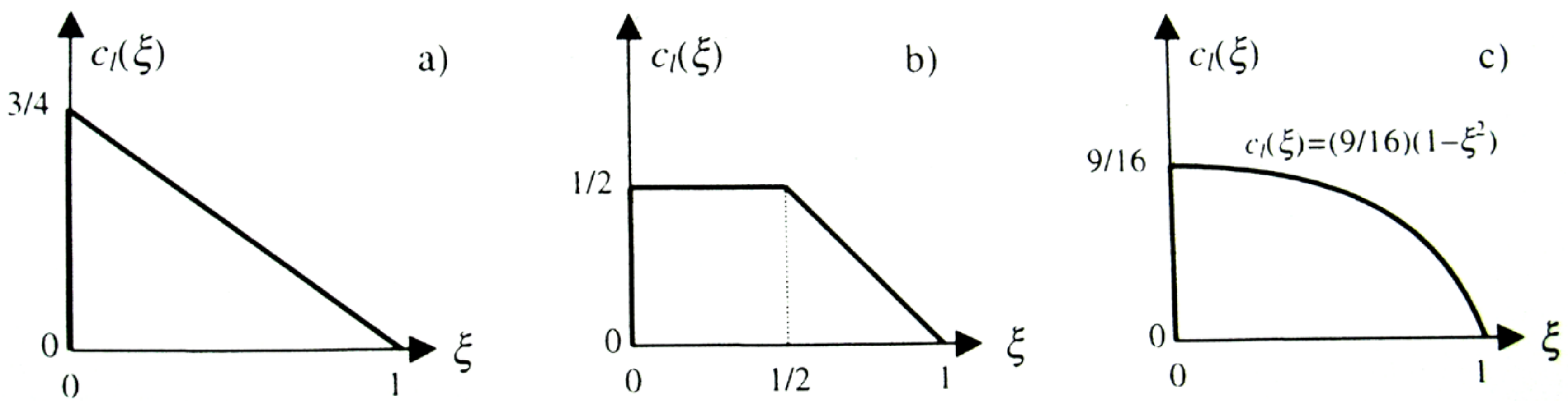
2. For the NACA 2412 airfoil, the lift coefficient and moment coefficient about the quarter-chord at -6° angle of attack are -0.39 and -0.045 respectively. At 4 ° angle of attack, these coefficients are 0.65 and -0.037. Calculate the location of the aerodynamic center.

Exercise

3. Considering the following figures of lift coefficients along the chord ($\xi = x/c$) of three different profiles with chord $c = 1.2m$.

   Calculate:

   - Total lift coefficient
   - Location of the center of pressure
   - Moment coefficient with respect to the aerodynamic center
   - Lift and moment of the profile flying at $80\ m/s$ with $\rho = 1.225\ Kg/m^3$



Exercise

4. Given a profile with chord c and whose camber lines is defined by:

$$z(x) = \varepsilon c\left[1 - 16\left(\frac{x}{c} + \frac{1}{4}\right)^2\right] \quad , \qquad -\frac{1}{2} \le \frac{x}{c} \le -\frac{1}{4}$$

$$z(x) = \varepsilon c\left[1 - \frac{16}{9}\left(\frac{x}{c} + \frac{1}{4}\right)^2\right] \quad , \qquad -\frac{1}{4} \le \frac{x}{c} \le \frac{1}{2}$$

with ε=0.02

Calculate:

- Ideal angle of attack (α_i)
- Angle of zero lift
- $c_{m(c/4)}$
- $c_l(\alpha=\alpha_i)$

Exercise

We can express the shape in using the angular variable

$$\begin{cases} z(x) = \varepsilon c\left[1 - 16\left(\dfrac{x}{c} + \dfrac{1}{4}\right)^2\right] \quad , \qquad -\dfrac{1}{2} \le \dfrac{x}{c} \le -\dfrac{1}{4} \\ z(x) = \varepsilon c\left[1 - \dfrac{16}{9}\left(\dfrac{x}{c} + \dfrac{1}{4}\right)^2\right] \quad , \qquad -\dfrac{1}{4} \le \dfrac{x}{c} \le \dfrac{1}{2} \end{cases}$$

$$\frac{x}{c} + \frac{1}{2} = \frac{1}{2}(1 - \cos\theta) \quad \rightarrow \quad \frac{x}{c} = -\frac{1}{2}\cos\theta$$

$$\frac{dz}{dx}\begin{cases} \dfrac{dz}{dx} = -32\varepsilon\left(\dfrac{x}{c} + \dfrac{1}{4}\right) = -8\varepsilon(1 - 2\cos\theta) \quad , \qquad 0 \le \theta \le \dfrac{\pi}{3} \\ \dfrac{dz}{dx} = -\dfrac{32}{9}\varepsilon\left(\dfrac{x}{c} + \dfrac{1}{4}\right) = -\dfrac{8}{9}\varepsilon(1 - 2\cos\theta) \quad , \qquad \dfrac{\pi}{3} \le \theta \le \pi \end{cases}$$

Exercise

And the coefficients are:

$$A_0 = \quad \alpha - \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx} d\theta = \alpha - \frac{1}{\pi}\left[\int_0^{\pi/3} \frac{dz}{dx} d\theta_0 + \int_{\pi/3}^{\pi} \frac{dz}{dx} d\theta_0\right] =$$

$$= \alpha - \frac{8\varepsilon}{\pi}\left[\int_0^{\pi/3}(1-2\cos\theta)d\theta + \frac{1}{9}\int_{\pi/3}^{\pi}(1-2\cos\theta)d\theta\right] = \alpha - \frac{8\varepsilon}{9\pi}\left(8\sqrt{3} - \frac{11}{3}\pi\right)$$

$$A_n = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos(n\theta)d\theta$$

$$A_1 = \quad \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos\theta d\theta = \frac{16\varepsilon}{\pi}\left[\int_0^{\pi/3}(1-2\cos\theta)\cos\theta d\theta + \frac{1}{9}\int_{\pi/3}^{\pi}(1-2\cos\theta)\cos\theta d\theta\right] =$$

$$= \frac{16\varepsilon}{9\pi}\left(\frac{11}{3}\pi - 2\sqrt{3}\right)$$

$$A_2 = \quad \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos(2\theta)d\theta = \frac{16\varepsilon}{\pi}\left[\int_0^{\pi/3}(1-2\cos\theta)\cos 2\theta d\theta + \frac{1}{9}\int_{\pi/3}^{\pi}(1-2\cos\theta)\cos 2\theta d\theta\right] =$$

$$= \frac{64\sqrt{3}\varepsilon}{9\pi}\left(1 - \frac{1}{2}\right)$$

Exercise

Thus the requested answers are:

- Ideal angle of attack (α_i) $\quad A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx} d\theta_0 = 0$

$$\alpha_i = \frac{8\varepsilon}{9\pi}\left(8\sqrt{3} - \frac{11}{3}\pi\right)$$

- Angle of zero lift ($\alpha_{L=0}$)

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}(\cos\theta_0 - 1)d\theta_0 = \underbrace{\alpha - A_0}_{\alpha_i} - \frac{1}{2}A_1 = \underbrace{\frac{8\varepsilon}{9\pi}\left(8\sqrt{3} - \frac{11}{3}\pi\right)}_{\alpha_i} - \frac{1}{2}\underbrace{\frac{16\varepsilon}{9\pi}\left(\frac{11}{3}\pi - 2\sqrt{3}\right)}_{A_1} =$$

$$= -\frac{16\varepsilon}{9\pi}\left(\frac{11}{3}\pi - 5\sqrt{3}\right)$$

- Moment coefficient at chord/4 ($c_{m(c/4)}$)

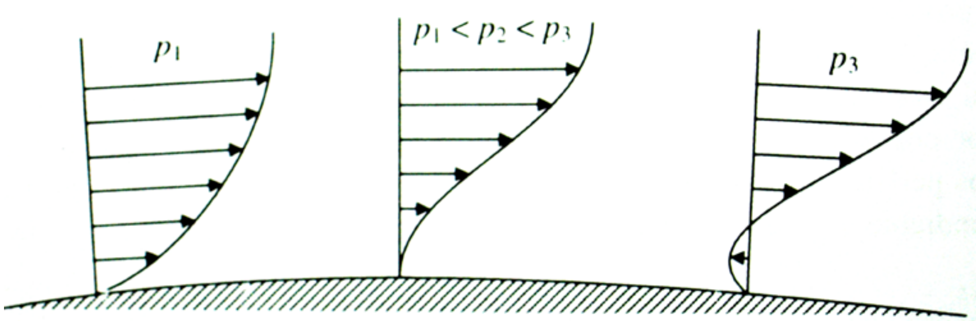
$$c_{m_{c/4}} = c_{m_{ac}} = \frac{\pi}{4}(A_2 - A_1) = 0.04$$

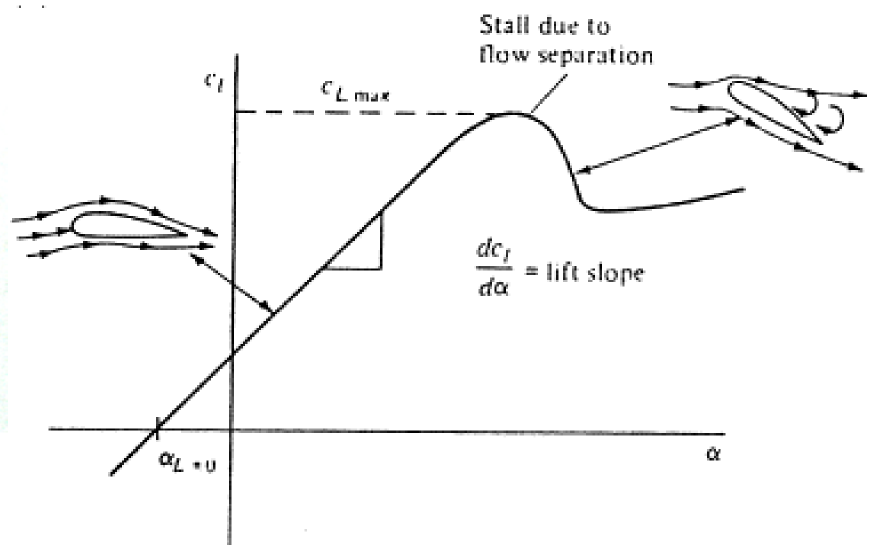
- Lift coefficient with the ideal angle of attack $c_l(\alpha=\alpha_i)$

$$c_l = \pi(2A_0 + A_1)$$

Airfoil boundary layer

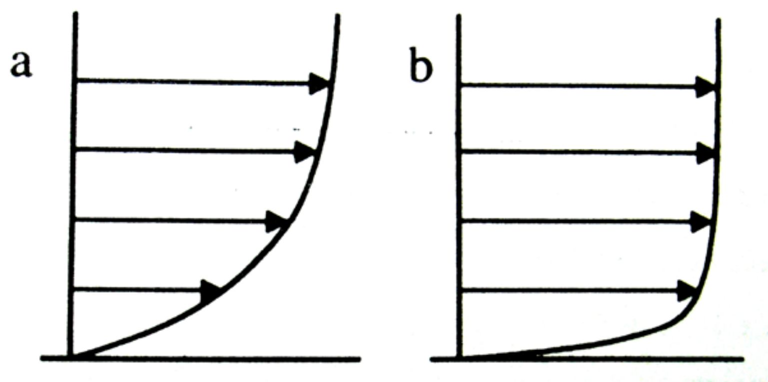
- The behavior of an airfoil close to the stall condition is governed by the influence of the boundary layer, which has the following important properties:
  - The pressure is constant along straight lines perpendicular to the profile.
  - The boundary layer thickness grows along the profile by two reasons:
    - The boundary layer is decelerated due to the viscous stress.
    - The momentum in the boundary layer is reduced to the need of advancing against an adverse pressure gradient.
  - The deceleration of the fluid due to an adverse pressure gradient is much more important close to the wall.
  - The boundary layer can be laminar or turbulent.





Airfoil: laminar vs. turbulent

- Laminar boundary layer:
  - The viscous stress is proportional to the derivative of the velocity, thus friction drag I much smaller
  - It is unstable. Small perturbations may trigger unstable modes and the turbulent regime will appear.
  - Low energy in the inner layers of the boundary layer.
- Turbulent boundary layer:
  - Higher viscous stress and friction drag.
  - It is a stable mode
  - Higher energy in the inner layers of the boundary layer, yield to a better behavior against adverse pressure gradients.



a) Laminar b) Turbulent

Airfoil: stall

- Stall mechanism:

  As much as we increase the angle of attack of an airfoil, as higher the negative suction peak will be in the upper surface close to the leading edge. But we need to recover the pressure in the stagnation point in the trailing edge, thus we will face an adverse pressure gradient that can trigger the boundary layer detachment.

- Boundary layer evolution:
  - It starts in the stagnation point.
  - It is laminar until the suction peak (developing with favorable pressure gradient)
  - After the suction peak there are many options:

  1. Laminar boundary layer detachment without further reattachment.
  2. Laminar boundary layer detachment with further reattachment.
  3. Transition to turbulent boundary layer with further detachment.
  4. Transition to turbulent boundary layer without further detachment.

  The suction peak depends mainly on the leading edge curvature. High curvature yields to high suction peaks, the extreme case is the flat plate, where the curvature is infinite and the suction peak as well. The opposite case is the cylinder, with a very moderated suction peak.

Airfoil: stall

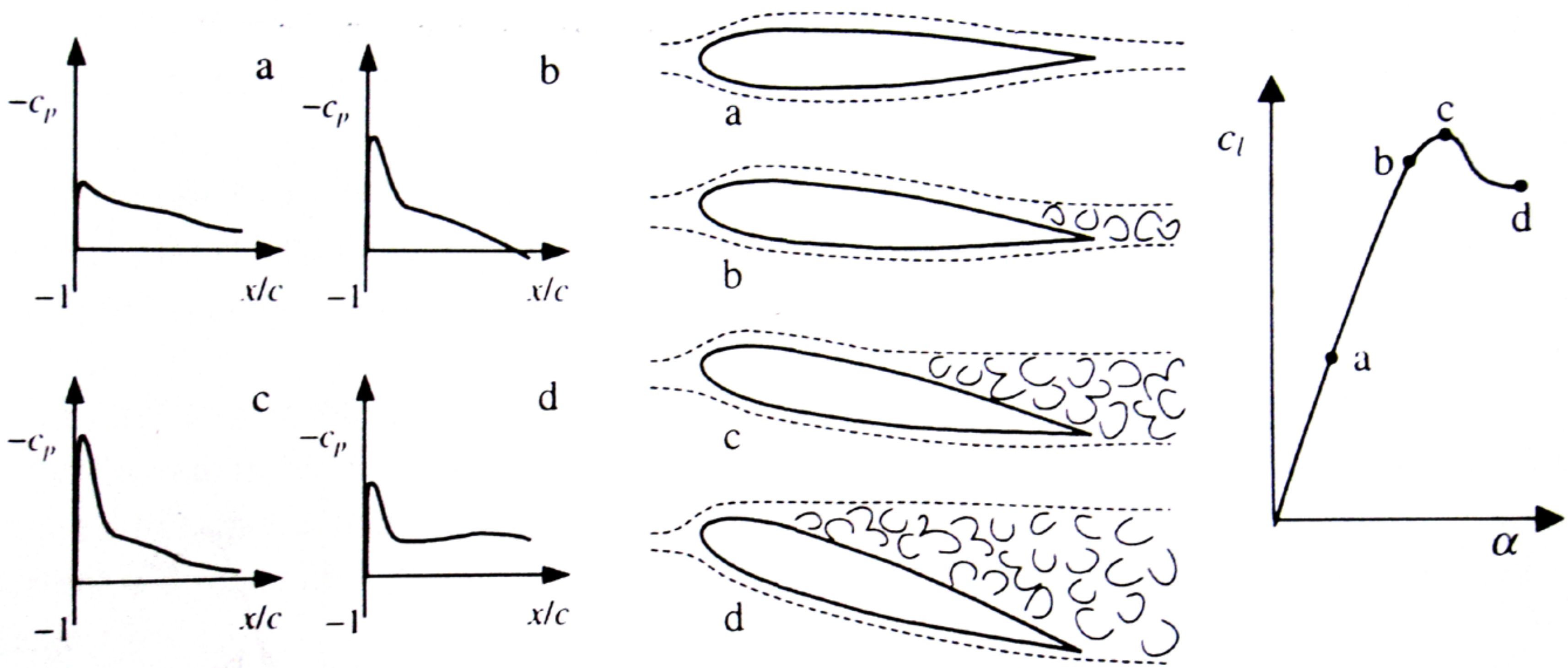
- Reynolds effect in airfoil stall
  - If the boundary layer transits to turbulent just after the detachment it may reattach after a recirculation bubble.
  - If we increase the Reynolds we move backwards the transition point and we can reduce the bubble size until it disappears.
- Airfoil stall characterization:

  The curvature in the leading edge fixes the suction peak, thus we can classify the airfoils based on their thickness and curvature:

  - Thick airfoils
  - Medium thickness airfoils
  - Thin airfoils

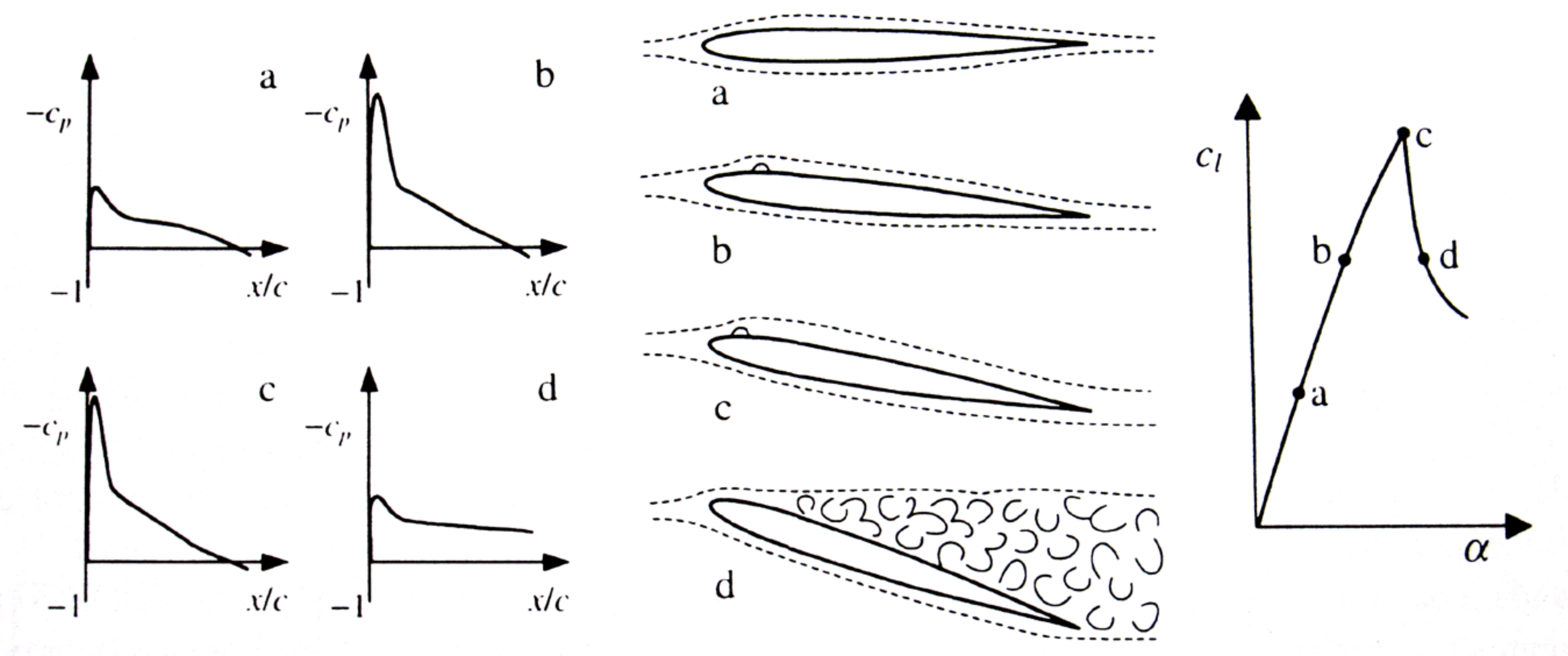
Airfoil: stall

- Thick airfoils
  - Detachment happens near the trailing edge



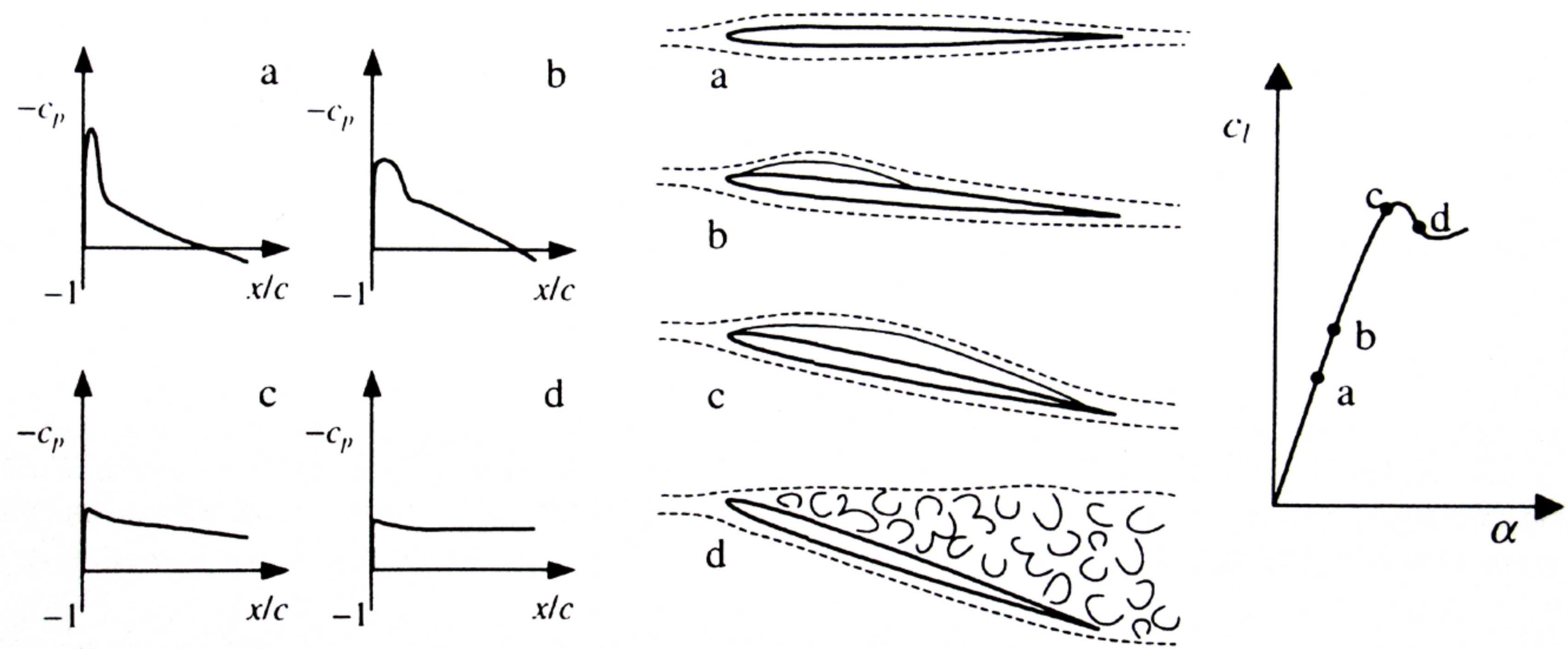
Airfoil: stall

- Medium thickness airfoils
  - The boundary layer detaches in laminar regime, but it transits quickly to turbulent and reattaches to the airfoil after a short bubble.
  - If we increase the angle of attack, the detachment point approaches the high curvature leading edge, and the bubble explodes suddenly.



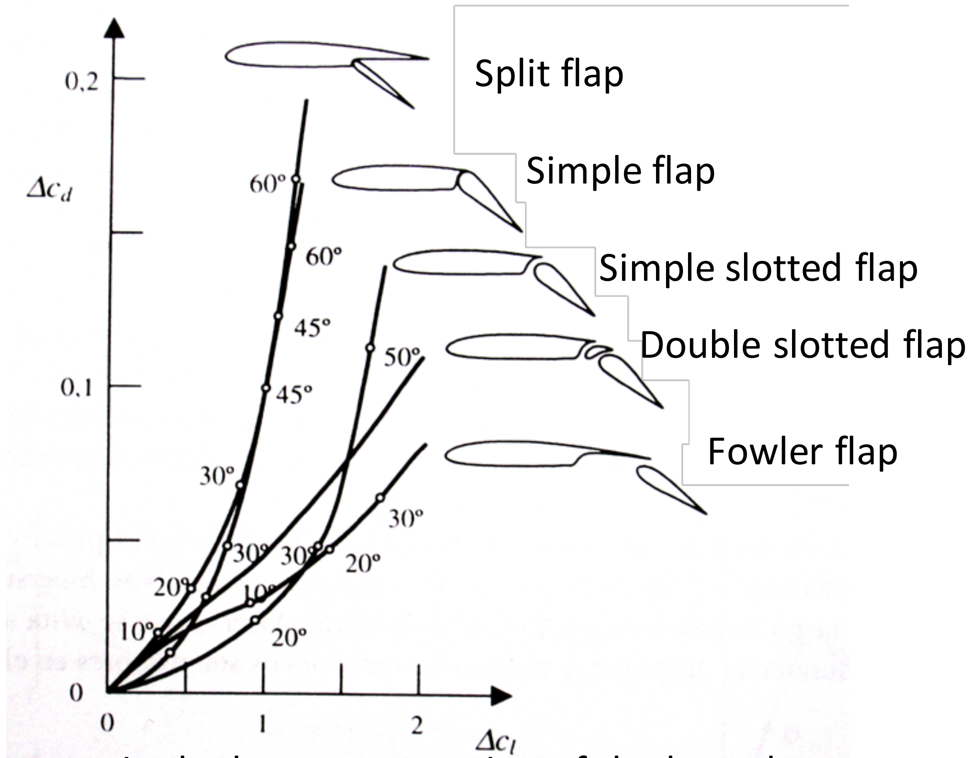
Airfoil: stall

- Thin airfoils
  - The boundary layer is very thin in the detachment point and it needs a certain distance to transit to turbulent.



Airfoil: high lift devices

- The typical aircraft mission requires very different values for c_l from 0.4 to 3.2
  - Cruise condition: minimum drag
  - Take-off: high lift and low drag
  - Landing: high lift and high drag



- How to increase maximum lift
  - Increase curvature
  - Increase lift surface (chord)
  - Boundary layer control: supply momentum in the low energy region of the boundary layer, or sucking this low energy boundary layer.
  - Decrease velocity on the suction peak (slat)
  - Energize the boundary layer (blow)

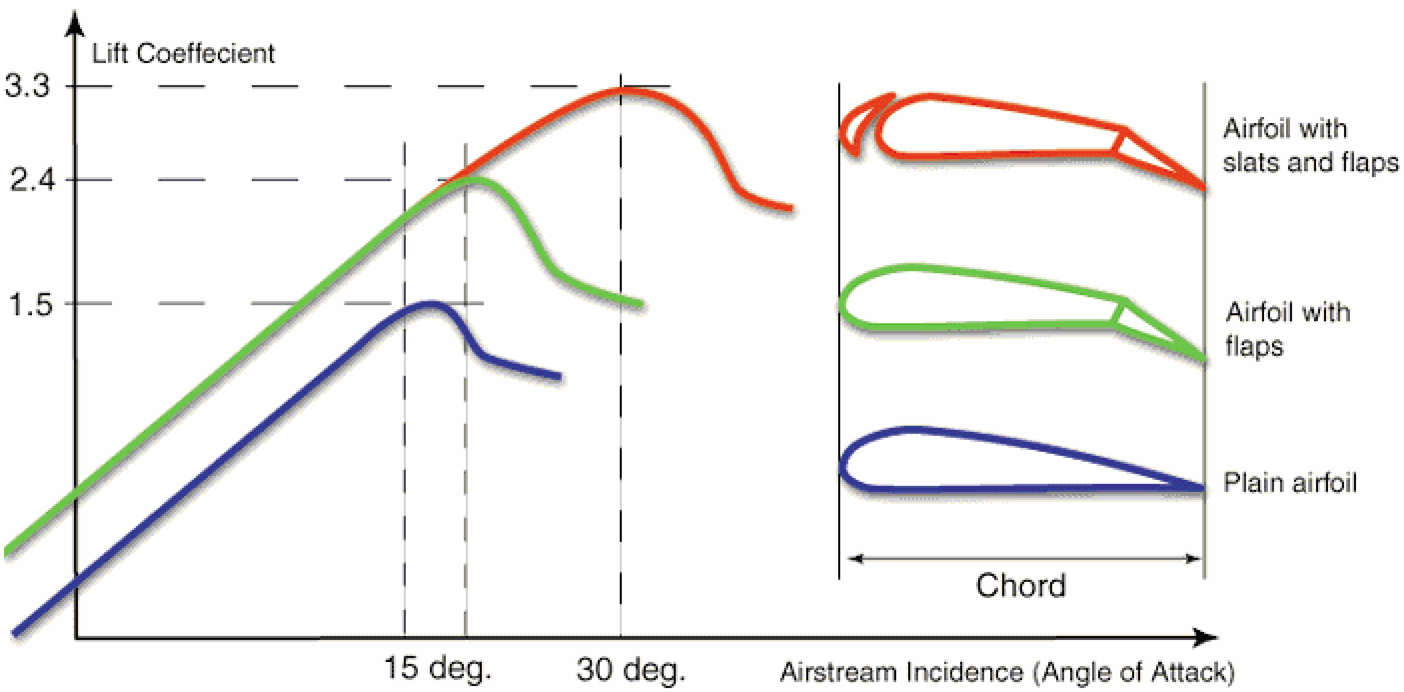
Airfoil: high lift devices

- Slat:

  It is a small profile deployed in front of the leading edge. Its contribution to the lift is almost neglectable, but its circulation reduces the flow speed in the leading edge.

- Slot:

  It is a fixed channel that connects the upper and lower surfaces in the leading edge.



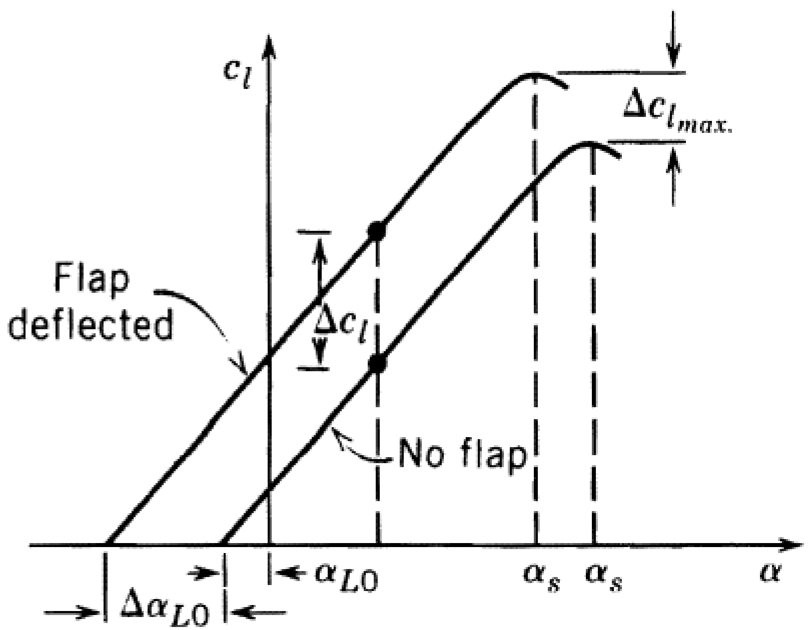
Airfoil: high lift devices

- The term ($\cos\theta$ -I) vanishes at the leading edge where $\theta = 0$; and its absolute value reaches maximum at the trailing edge where $\theta = \pi$. Thus, the portion of the mean camber line in the vicinity of the trailing edge powerfully influences the value of $\alpha_{L=0}$.

- It is on this fact that the aileron as a lateral-control device and the flap as a high-lift device are based. A deflection downward of a portion of the chord at the trailing edge effectively makes the ordinates of the mean camber line more positive in this region.

- $\alpha_{L=0}$LO becomes more negative and the lift at a given geometric angle of attack is increased.

Airfoil: high lift devices



Effect of flap deflection on lift curve.

The gain in lift at the given geometric angle of attack is shown as ΔCl
If the rear portion of the trailing edge is deflected upward, an opposite displacement of the lift curve results and the lift at a given geometric angle of attack is decreased.

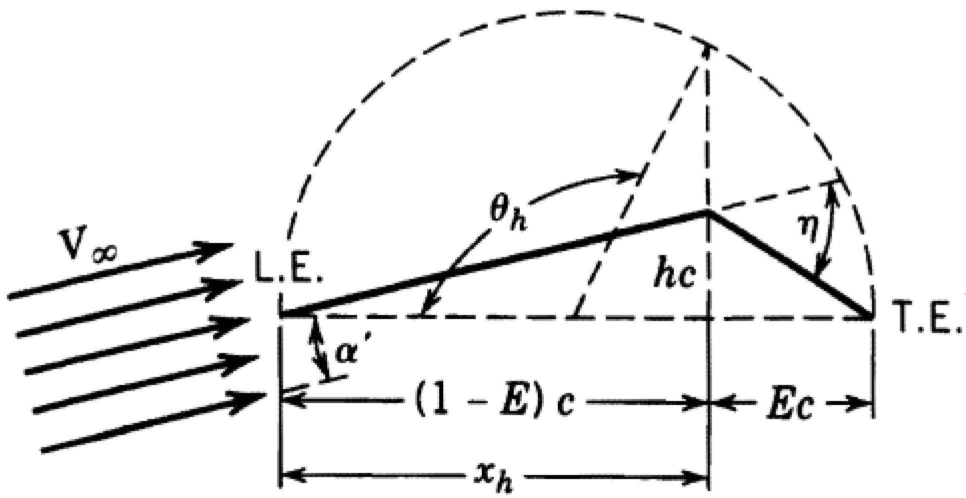
Airfoil: high lift devices

Because all angles are small, it is sufficient to find the properties of a symmetrical airfoil at zero angle of attack with flap deflected.

These may be added directly to the properties of the cambered airfoil at any angle of attack.

If the leading and trailing edges are connected by a straight line and if this is treated as a fictitious chord line. the problem reduced to that of a cambered airfoil at an angle of attack α'

Airfoil: high lift devices



$$A_0 = \alpha' - \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}\,d\theta$$

$$A_n = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos n\theta\,d\theta$$

The integrals must be evaluated in two parts: from the leading edge to the hinge line θ_c and from the hinge line to the trailing edge. Ao becomes

$$A_0 = \alpha' - \frac{1}{\pi}\left[\frac{h}{1-E}\theta\right]_0^{\theta_h} - \frac{1}{\pi}\left[-\frac{h}{E}\theta\right]_{\theta_h}^{\pi}$$

Airfoil: high lift devices

After we substitute in the limits and use the relations

$$\frac{h}{1-E}+\frac{h}{E} = \eta$$

$$\alpha'+\frac{h}{E} = \eta$$

the value of *Ao* becomes

$$A_0 = \frac{\eta(\pi-\theta_h)}{\pi}$$

In a similar manner, the values of A_n are found to be

$$A_n = \frac{2\eta\sin n\theta_h}{n\pi}$$

Airfoil: high lift devices

These equations, when substituted into Equations yield incremental aerodynamic characteristics ΔC_l and Δc_{mac} due to the flap deflection:

$$\Delta c_l = 2[(\pi - \theta_h) + \sin\theta_h]\,\eta$$

$$\Delta c_{mac} = [\tfrac{1}{2}\sin\theta_h(\cos\theta_h - 1)]\,\eta$$

$$\Delta\alpha_{L0} = -\Delta c_l/2\pi = -[(\pi - \theta_h) + \sin\theta_h]\,\eta/\pi$$

These equations show that the incremental values of ΔCl, Δcmac and $\alpha_{L=0}$.vary linearly with the flap deflection.

The magnitudes are shown to be strong functions of θ_h , which is related to the distance X_h of the hinge line behind the leading edge by the expression

$$x_h = \tfrac{1}{2}c(1 - \cos\theta_h)$$

Airfoil: high lift devices

These equations, when substituted into Equations yield incremental aerodynamic characteristics ΔC_l and Δc_{mac} due to the flap deflection:

$$\Delta c_l = 2[(\pi - \theta_h) + \sin\theta_h]\,\eta$$

$$\Delta c_{mac} = [\tfrac{1}{2}\sin\theta_h(\cos\theta_h - 1)]\,\eta$$

$$\Delta\alpha_{L0} = -\Delta c_l/2\pi = -[(\pi - \theta_h) + \sin\theta_h]\,\eta/\pi$$

These equations show that the incremental values of ΔCl, Δcmac and $\alpha_{L=0}$.vary linearly with the flap deflection.

The magnitudes are shown to be strong functions of θ_h , which is related to the distance X_h of the hinge line behind the leading edge by the expression

$$x_h = \tfrac{1}{2}c(1 - \cos\theta_h)$$

Airfoil: high lift devices