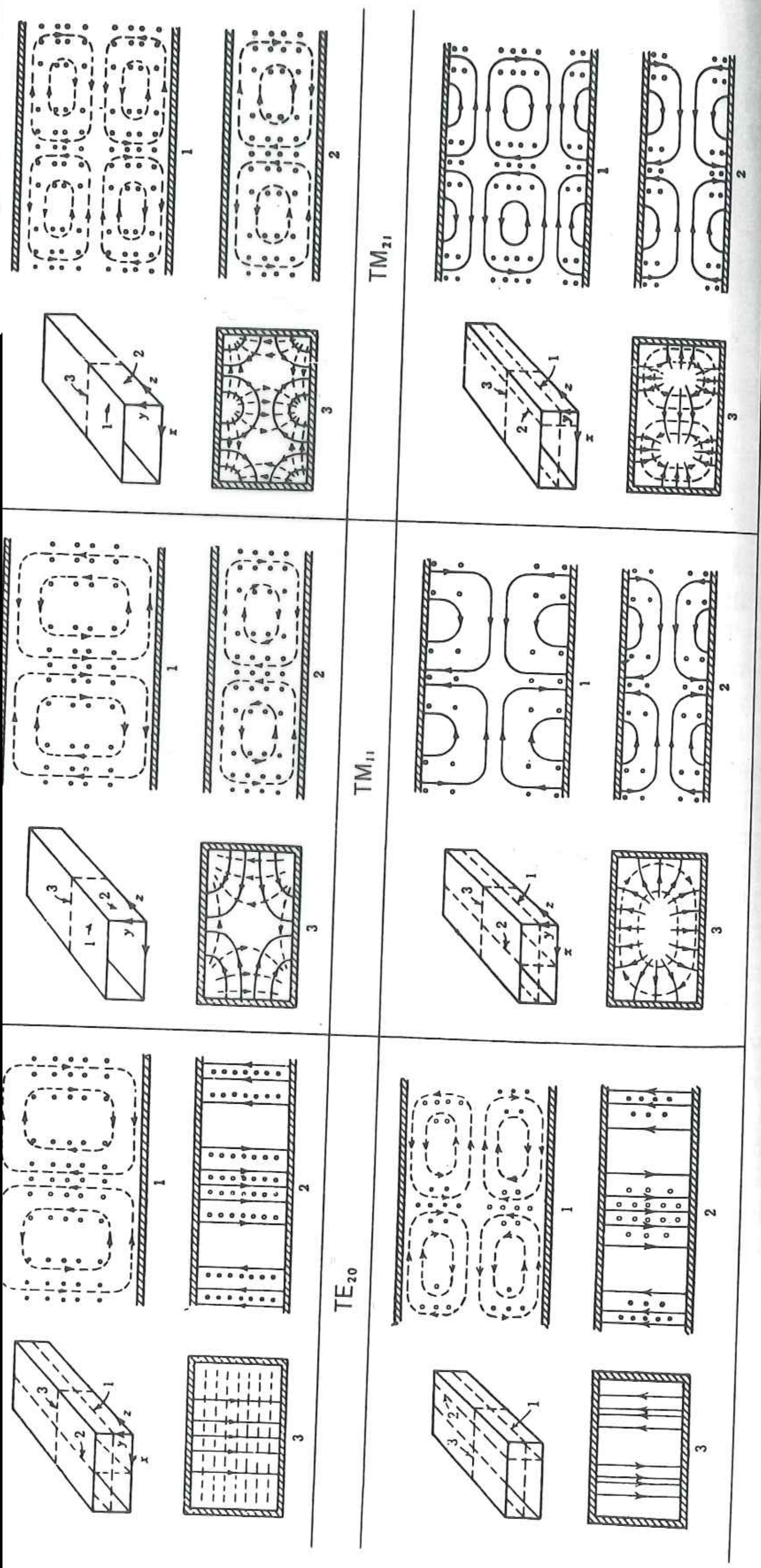
…-3
…y of $TE^z_{mn}$ and $TM^z_{mn}$ mode characteristics of rectangular waveguide

| $TE^z_{mn}\left(\begin{matrix} m = 0, 1, 2, \ldots \\ n = 0, 1, 2, \ldots \end{matrix}\quad m = n \neq 0\right)$ | $TM^z_{mn}\left(\begin{matrix} m = 1, 2, 3, \ldots \\ n = 1, 2, 3, \ldots \end{matrix}\right)$ |
|---|---|
| $A_{mn}\frac{n\pi}{b\varepsilon}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_z z}$ | $-B_{mn}\frac{m\pi\beta_z}{a\omega\mu\varepsilon}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_z z}$ |
| $-A_{mn}\frac{m\pi}{a\varepsilon}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_z z}$ | $-B_{mn}\frac{n\pi\beta_z}{b\omega\mu\varepsilon}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_z z}$ |
| $0$ | $-jB_{mn}\frac{\beta_c^2}{\omega\mu\varepsilon}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_z z}$ |
| $A_{mn}\frac{m\pi\beta_z}{a\omega\mu\varepsilon}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_z z}$ | $B_{mn}\frac{n\pi}{b\mu}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_z z}$ |
| $A_{mn}\frac{n\pi\beta_z}{b\omega\mu\varepsilon}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_z z}$ | $-B_{mn}\frac{m\pi}{a\mu}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_z z}$ |
| $-jA_{mn}\frac{\beta_c^2}{\omega\mu\varepsilon}\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_z z}$ | $0$ |
| $\sqrt{\beta_x^2 + \beta_y^2} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$ | |
| $\frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$ | |
| $\frac{2\pi}{\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$ | |
| $\beta\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$ | |
| $\frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$ | |
| $\frac{v}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{v}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$ | |
| $\frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\eta}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$ | $\eta\sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \eta\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}$ |
| $j\frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = j\frac{\eta}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$ | $-j\eta\sqrt{1 - \left(\frac{f_c}{f}\right)^2} = -j\eta\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}$ |

| | $TE^z_{mn}$ (… $= 0, 1, 2, \ldots$; … $= 0, 1, 2, \ldots$; $m = n \neq 0$) | $TM^z_{mn}\left(\begin{matrix} m = 1, 2, 3, \ldots \\ n = 1, 2, 3, \ldots \end{matrix}\right)$ |
|---|---|---|
| $(\alpha_c)_{mn}$ | … $\left\{\left(\varepsilon_m + \varepsilon_n\frac{b}{a}\right)\left(\frac{f_{c,mn}}{f}\right)^2$ … $\left.\right]\frac{m^2ab + (na)^2}{(mb)^2 + (na)^2}\right\}$ | $\frac{2R_s}{ab\eta\sqrt{1 - \left(\frac{f_{c,mn}}{f}\right)^2}}\frac{m^2b^3 + n^2a^3}{(mb)^2 + (na)^2}$ |
| | … $= \begin{cases} 2 & p = 0 \\ 1 & p \neq 0 \end{cases}$ | |




[a] Electric field lines are shown solid and magnetic field lines are dashed.



Solution by the separation

$$H_z = (A'' \sin$$

where

Imposition of boundary co
and 8.2(14) we find electr

$$E_x = -\frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}$$

$$= -\frac{j\omega\mu k_y}{k_c^2} \ (.$$

$$E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}$$

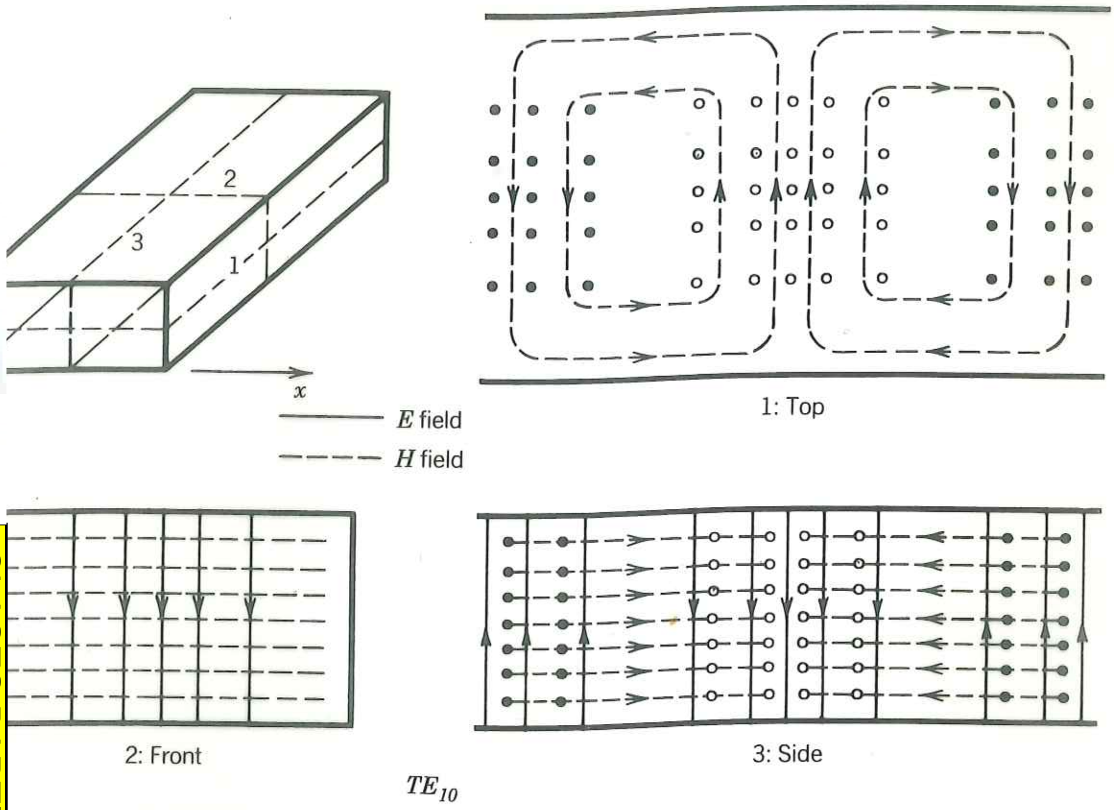
$$= \frac{j\omega\mu k_x}{k_c^2} (A''$$

For $E_x$ to be zero at $y$ =
$A'' = 0$. Defining $B''D''$

We also require $E_x$ to be
at $x = a$ so that $k_x a$ is a

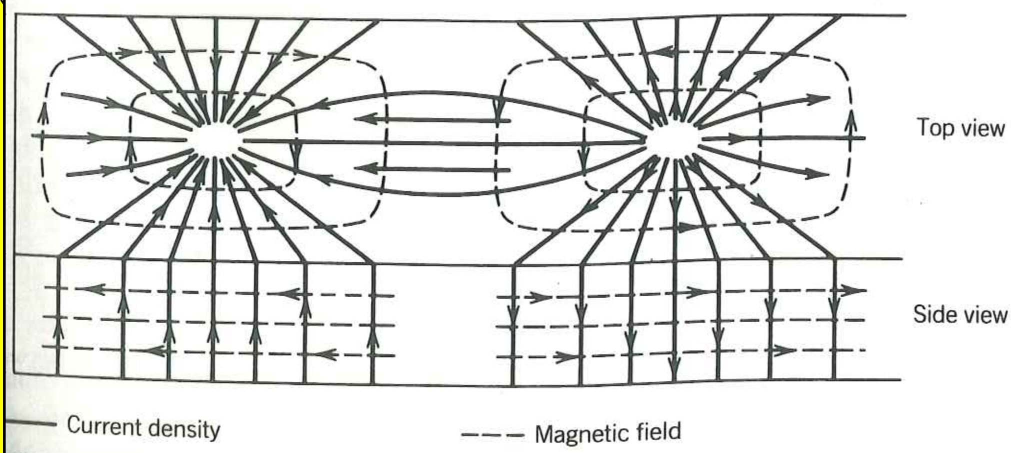
In contrast to the TM v
wave's vanishing. Altho
tric field, we can see fro
normal to the conductin
zero there, so boundary
requiring the explicit fo

The forms of transv
(19) are




Electric field patterns for $TE_{10}$ mode in a rectangular waveguide (*Source:* S. Ramo, J. R. …innery, and T. Van Duzer, *Fields and Waves in Communication Electronics*, 1984. Reprinted … permission of John Wiley & Sons, Inc.)

From the preceding information it is evident that the electric field intensity …de the guide has only one component, an $E_y$. The $E$- and $H$-field variations on … top, front, and side views of the guide are shown graphically in Figure 8-5, and … current density and $H$-field lines on the top and side views are shown in Figure … [2]. It is instructive at this time to examine the electric field intensity a little …er and attempt to provide some physical interpretation of the propagation




…RE 8-6 Magnetic field and electric current density patterns for the $TE_{10}$ mode in a rectangular waveguide. (*Source:* S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics*, 1984. Reprinted with permission of John Wiley & Sons, Inc.)

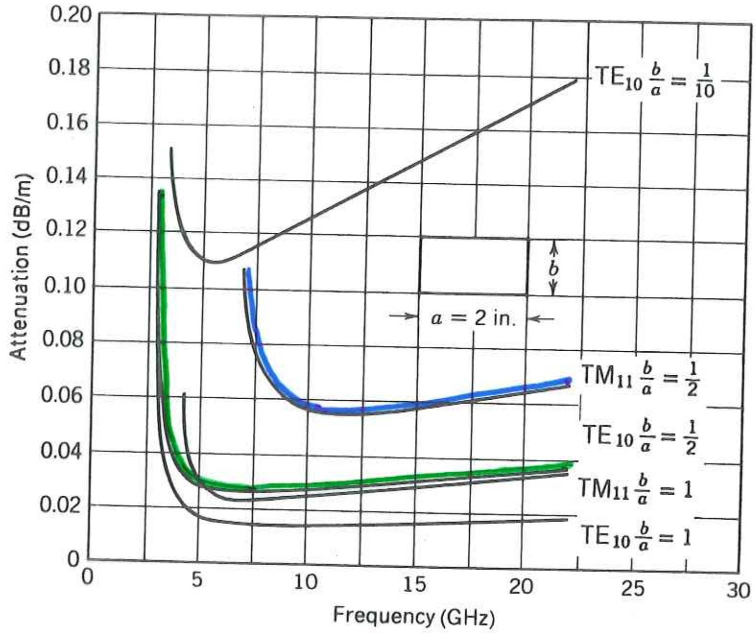



**FIG. 8.7c** Attenuation due to copper losses in rectangular waveguides of fixed width.

The attenuation constant for $TE_{mn}$ $(n \neq 0)$ modes is found using power transfer and power loss per unit length as in Eq. 8.5(11).

$$(\alpha_c)_{TE_{mn}} = \frac{2R_s}{b\eta\sqrt{1 - (f_c/f)^2}} \left\{ \left(1 + \frac{b}{a}\right)\left(\frac{f_c}{f}\right)^2 + \left[1 - \left(\frac{f_c}{f}\right)^2\right]\left[\frac{(b/a)((b/a)m^2 + n^2)}{(b^2m^2/a^2) + n^2}\right]\right\} \tag{26}$$

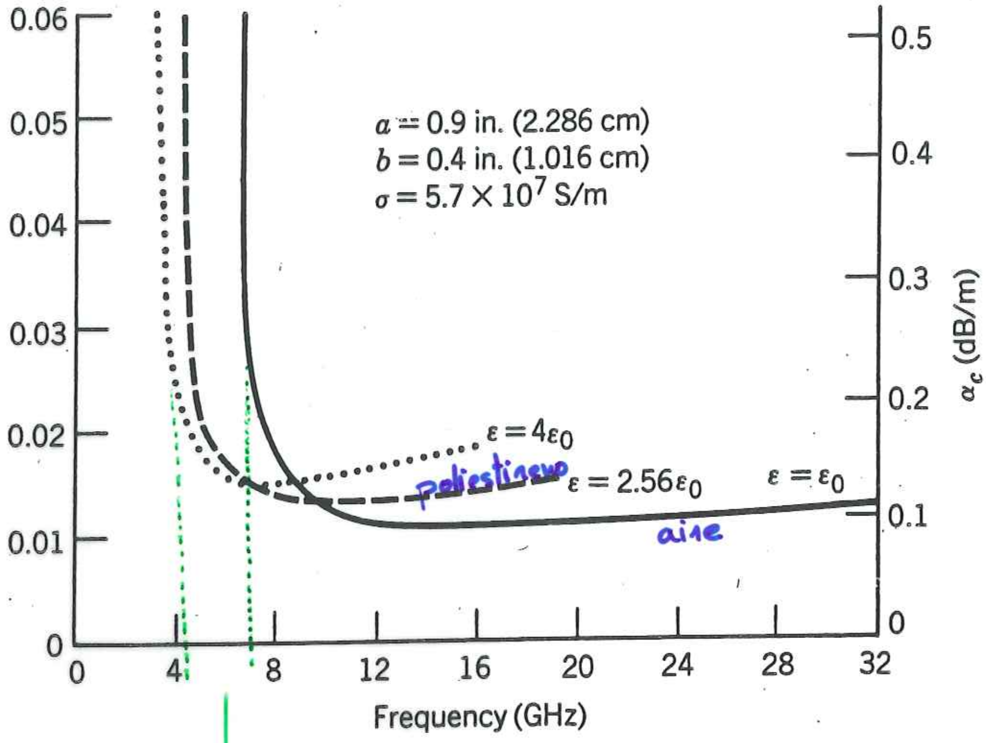
And for $TE_{m0}$ modes

$$(\alpha_c)_{TE_{m0}} = \frac{R_s}{b\eta\sqrt{1 - (f_c/f)^2}} \left[1 + \frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right] \tag{27}$$

Figure 8.7$c$ shows attenuation versus frequency for $TM_{11}$ and $TE_{10}$ modes in rectangular copper waveguides with various side ratios $b/a$ found using (14) and (27), respectively. It is seen that small $b/a$ ratios give large attenuations because of the high ratio of surface to cross-sectional area.

## 8.8 THE $TE_{10}$ WAVE IN A RECTANGULAR GUIDE

One of the simplest of all the waves which may exist inside hollow-pipe waveguides is the dominant $TE_{10}$ wave in the rectangular guide, which is one of the TE modes




GURE 8-10 $TE_{10}$ mode attenuation constant for the $X$-band rectangular waveguide.

frecuencias de corte disminuyen cuanto más "dieléctrico" es el interior de la guía.