$$\phi(x,t) = \frac{1}{(2\pi\sigma^2)^{1/4}} \frac{1}{\sqrt{2\pi\hbar}} e^{-\frac{p_0^2}{4\sigma^2}} \int_{-\infty}^{+\infty} dp \; e^{-\overbrace{\left[\frac{1}{4\sigma^2} + \frac{it}{2m\hbar}\right]}^{\alpha^2} p^2 - p \overbrace{\left[-\frac{p_0}{2\sigma^2} - \frac{ix}{\hbar}\right]}^{\gamma}}$$

$$= \frac{1}{(2\pi\sigma^2)^{1/4}} \frac{1}{\sqrt{2\pi\hbar}} e^{-\frac{p_0^2}{4\sigma^2}} \frac{\sqrt{\pi}}{\sqrt{\frac{1}{4\sigma^2} + \frac{it}{2m\hbar}}} \; e^{\left(\frac{p_0}{2\sigma^2} + \frac{ix}{\hbar}\right)^2 \Big/ 4\left(\frac{1}{4\sigma^2} + \frac{it}{2m\hbar}\right)}$$

$$|\phi(x,t)|^2 = \frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{2\hbar} \frac{e^{-p_0^2/2\sigma^2}}{\sqrt{\left(\frac{1}{4\sigma^2}\right)^2 + \left(\frac{t}{2m\hbar}\right)^2}} \; e^{-2\mathrm{Re}\left[\frac{\left(\frac{p_0}{4\sigma^2} + \frac{ix}{\hbar}\right)^2}{4\left(\frac{1}{4\sigma^2} + \frac{it}{2m\hbar}\right)}\right]}$$

$$= \sqrt{\frac{\sigma^2}{\hbar^2}} \left(\frac{2}{\pi}\right)^{1/4} \frac{\cancel{e^{-p^2/2\sigma^2}}}{\sqrt{1 + \left(\frac{4\sigma^2 t}{2m\hbar}\right)^2}} \left\{ e^{-\frac{2\sigma^2}{1 + \frac{4(\sigma^2)^2 t^2}{m^2\hbar^2}} \left(x - \frac{p_0 t}{m}\right)^2 / \hbar^2} \times \cancel{e^{+p_0^2/2\sigma^2}} \right\}$$

$$|\phi(x,t)|^2 = \sqrt{\frac{\sigma^2}{\hbar^2}} \left(\frac{2}{\pi}\right)^{1/4} \frac{1}{\sqrt{1 + \frac{4(\sigma^2)^2 t^2}{m^2\hbar^2}}} \; e^{-\frac{2\sigma^2}{1 + \frac{4(\sigma^2)^2 t^2}{m^2\hbar^2}} \left(x - \frac{p_0 t}{m}\right)^2 / \hbar^2}$$

$$2\,\mathrm{Re}\left[\frac{\left(\frac{p_0}{4\sigma^2} + \frac{ix}{\hbar}\right)^2}{4\left(\frac{1}{4\sigma^2} + \frac{it}{2m\hbar}\right)}\right] = \frac{1}{2}\,\mathrm{Re}\left\{\frac{\left(\frac{p_0}{2\sigma^2} + \frac{ix}{\hbar}\right)^2 \left(\frac{1}{4\sigma^2} - \frac{it}{2m\hbar}\right)}{\left(\frac{1}{4\sigma^2}\right)^2 + \frac{t^2}{(2m\hbar)^2}}\right\} =$$

$$= \mathrm{Re}\left\{ 2\sigma^2 \frac{\left(\frac{p_0}{2\sigma^2} + \frac{ix}{\hbar}\right)^2 \left(\frac{1}{4\sigma^2} - \frac{it}{2m\hbar}\right)}{1 + \left(\frac{4\sigma^2 t}{2m\hbar}\right)^2} \right\} =$$

$$= \frac{2\sigma^2}{1 + \frac{4(\sigma^2)^2 t^2}{m^2\hbar^2}} \left[\left(\frac{p_0}{2\sigma^2}\right)^2 - \frac{x^2}{\hbar^2} - \frac{2p_0 x}{\cancel{2\sigma^2}\hbar} \frac{4\sigma^2 t}{\cancel{2}m\hbar} \right] =$$

$$= \frac{2\sigma^2}{1+\frac{4(\sigma^2)^2t^2}{m^2\hbar^2}}\left[\left(\frac{p_0}{2\sigma^2}\right)^2 - \frac{x^2}{\hbar^2} + \frac{2x}{\hbar^2}\frac{p_0 t}{m}\right] =$$

completando cuadrado

$$= \frac{2\sigma^2}{1+\frac{4(\sigma^2)^2t^2}{m^2\hbar^2}}\left[\left(\frac{p_0}{2\sigma^2}\right)^2 - \frac{1}{\hbar^2}\left(x-\frac{p_0 t}{m}\right)^2 + \frac{p_0^2t^2}{m^2\hbar^2}\right]$$

$$= \frac{2\sigma^2}{1+\frac{4(\sigma^2)^2t^2}{m^2\hbar^2}}\left(x-\frac{p_0t}{m}\right)^2\Big/\hbar^2 + \frac{2\sigma^2}{1+\frac{4(\sigma^2)^2t^2}{m^2\hbar^2}}\left[\frac{p_0^2}{4(\sigma^2)^2} + \frac{p_0^2t^2}{m^2\hbar^2}\right]$$

$$= \frac{2\sigma^2}{1+\frac{4(\sigma^2)^2t^2}{m^2\hbar^2}}\left(x-\frac{p_0t}{m}\right)^2\Big/\hbar^2 + p_0^2\frac{2\sigma^2}{4(\sigma^2)^2}\underbrace{\left(\frac{\left[1+\frac{4(\sigma^2)^2t^2}{m^2\hbar^2}\right]}{\left[1+\frac{4(\sigma^2)^2t^2}{m^2\hbar^2}\right]}\right)}_{\to 1} =$$

$$= \frac{2\sigma^2}{1+\frac{4(\sigma^2)^2t^2}{m^2\hbar^2}}\left(x-\frac{p_0t}{m}\right)^2\Big/\hbar^2 + p_0^2/2\sigma^2$$

$$P(t-t_0) = \frac{1}{\sqrt{2\pi\sigma_s^2}}\, e^{-\frac{(t-t_0)^2}{2\sigma_s^2}}$$

$$|\phi(x,t)|^2 = \frac{1}{\sqrt{2\pi(\Delta x(t))^2}}\, e^{-\left(x-\frac{p_0t}{m}\right)^2/2(\Delta x(t))^2}$$

$$\Delta x(t) = \frac{\hbar}{2\sigma}f(t);\quad f(t) = \sqrt{1+\frac{4(\sigma^2)^2t^2}{m^2\hbar^2}}$$

$$|\tilde{\phi}(p,t)|^2 = |\tilde{\phi}(p)|^2 = \frac{1}{\sqrt{2\pi(\Delta p(t))^2}}\, e^{-\frac{(p-p_0)^2}{2(\Delta p(t))^2}};\quad \Delta p(t) = \sigma$$

$$\Delta x(t)\,\Delta p(t) = \frac{\hbar}{2\sigma}f(t)\cdot\sigma = \frac{\hbar}{2}f(t) \geq \hbar$$