PROBLEMS – BLOCK II – PART ONE

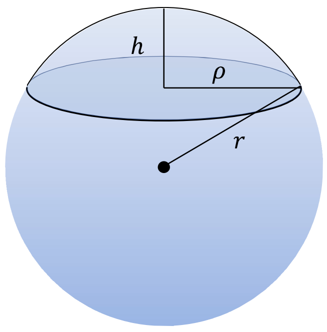
PART I - Monte Carlo. Integration.

Exercise 1 [**October 26, 2016**] A spherical cup is obtained when a sphere is cut by a plane. Consider the cap in the figure: the sphere has a radius $r = 1$ and is centered in the origin (0,0,0). The height h of the cap is $h = 0.6$ (ρ is not relevant).

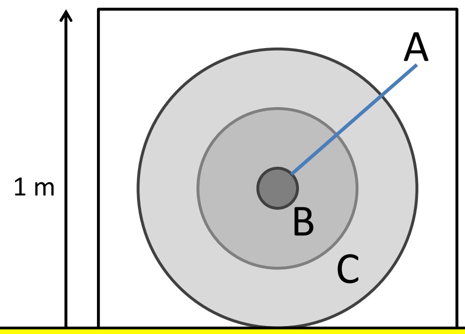
1. Write a probabilistic code to calculate the volume of the spherical cap. This code has to store the volume for $M = 400$ runs. Each run must calculate the volume using $N = 1000$ points.

2. Obtain the following statistical variables from the list of M volumes: mean value, range, median and standard deviation. Which values would you use to provide the best approximation to the volume of the cap?

3. Plot a 9-bin histogram and find (approximately) the number of cases with a volume between 0.8 and 1.0. What is its associated probability $P(0.8 < volume < 1.0)$?



Exercise 2 [**Final Exam: January 13, 2015**] When playing darts in a certain bar, we know that all darts hit a square area with dimensions $1m \times 1m$. In the center of this square we have a simplified dartboard divided into three zones: A (radius $r = 5\ cm$), B (radius $r = 20\ cm$) and C (radius $r = 40\ cm$). Each times a dart hits zone A, the player receives 100 points, while zone B and C are rewarded with 20 and 10 points, respectively. Shots outside the dartboard get 0 points. A one-player game consists of 10 throws. We want to simulate a large number of games, N. If all points inside the square have the same hit probability:



Exercise 3 [**November 13, 2013**] One of the purposes of Monte Carlo methods is to calculate multidimensional integrals. For the moment, we will restrict ourselves to $\mathbb{R}^6$, the Euclidean 6-dimensional space.

- Use 1000 random numbers to estimate the volume of the closed unit ball in $\mathbb{R}^6$.
- Explain how your approximation could be improved and provide a more accurate volume.
- If the relative error of the volume decreases with the number of trial as the inverse of the standard deviation of a random walk, find the number of trials needed to approximate the volume within a 1% of accuracy.
- Find a general formula for the volume of the closed unit ball in $\mathbb{R}^6$. *Hint: as you can guess, an integer power of* π *is present in the formula*.

Do not forget to provide a copy of your code. Make sure that it is correctly indented.