



CEU

*Universidad
San Pablo*

UNIT I – Review of Statistics: Jointly normal variables

Cartagena99

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP:689 45 44 70

Jointly Gaussian random variables

- X and Y are jointly Gaussian r.v., is they are normal and the joint p.d.f. is

$$f_{XY}(x, y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left(\frac{(x-\mu_X)^2}{\sigma_X^2} - \frac{2\rho(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y} + \frac{(y-\mu_Y)^2}{\sigma_Y^2}\right)},$$
$$-\infty < x < +\infty, \quad -\infty < y < +\infty, \quad |\rho| < 1.$$

where ρ is the correlation coefficient of X and Y

Cartagena99

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP:689 45 44 70

Jointly Gaussian random variables

- If X and Y are uncorrelated, then $\rho = 0$

$$\begin{aligned}
 f_{XY}(x, y) &= \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left(\frac{(x-\mu_X)^2}{\sigma_X^2} - \frac{2\rho(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y} + \frac{(y-\mu_Y)^2}{\sigma_Y^2}\right)} \\
 &= \frac{1}{2\pi\sigma_X\sigma_Y} e^{-\frac{1}{2}\left(\frac{(x-\mu_X)^2}{\sigma_X^2} + \frac{(y-\mu_Y)^2}{\sigma_Y^2}\right)} \\
 &= \frac{1}{\sqrt{2\pi\sigma_X^2}\sqrt{2\pi\sigma_Y^2}} e^{-\frac{(x-\mu_X)^2}{2\sigma_X^2}} e^{-\frac{(y-\mu_Y)^2}{2\sigma_Y^2}} = f_X(x)f_Y(y)
 \end{aligned}$$

X and Y are independent!

Cartagena99

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
 LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
 CALL OR WHATSAPP: 689 45 44 70

Jointly Gaussian random variables

- Normal r.v. are completely defined by their **mean** and **variance**
- Two **uncorrelated** jointly gaussian r.v. are **independent**
- Any **linear transformation** of several normal r.v. leads to a normal r.v.
- Given a jointly Gaussian p.d.f., any **marginal** function will be normal

Cartagena99

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP:689 45 44 70