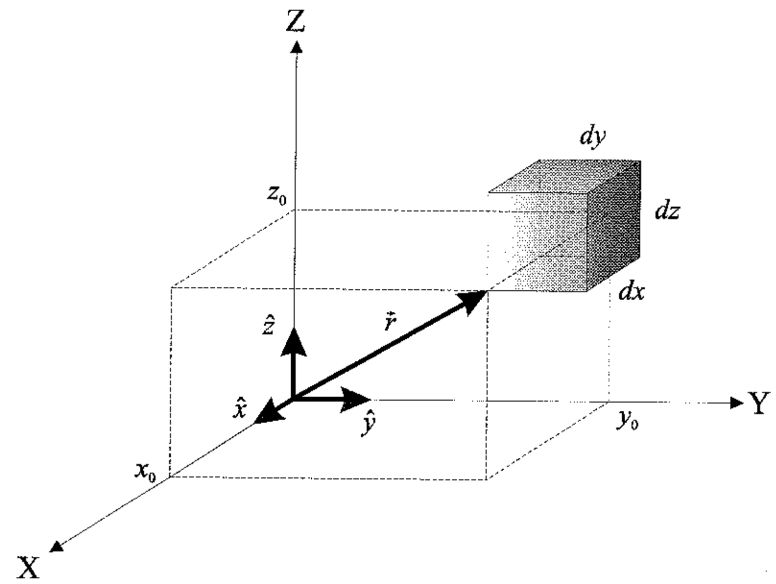
Anexo A: Sistemas de coordenadas

Coordenadas rectangulares

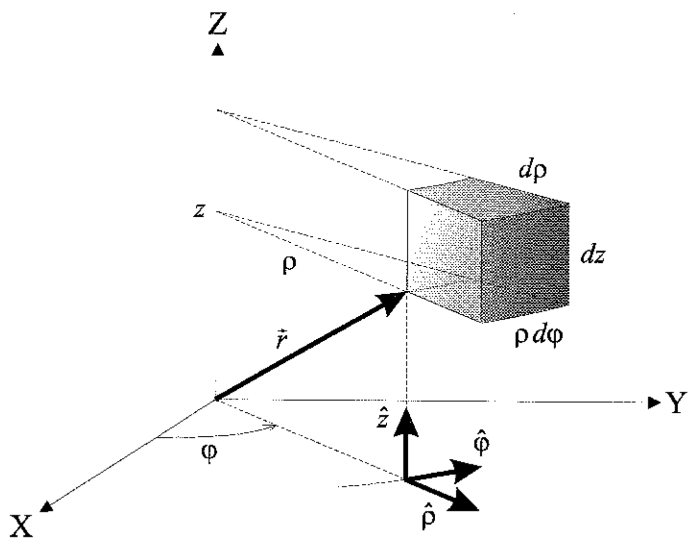


- radiovector desde el origen: $\vec{r} = x_0\hat{x} + y_0\hat{y} + z_0\hat{z}$

- diferencial de longitud genérico: $d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$
- diferenciales de superficie principales: $d\vec{s} = dx\,dy\,\hat{z},\ dx\,dz\,\hat{y},\ dy\,dz\,\hat{x}$
- diferencial de volumen: $dv = dx\,dy\,dz$

Los vectores unitarios principales son $\hat{x}, \hat{y}, \hat{z}$ con las relaciones: $\hat{x}\times\hat{y}=\hat{z},\ \hat{z}\times\hat{x}=\hat{y},\ \hat{y}\times\hat{z}=\hat{x}$.

Coordenadas cilíndricas



- radiovector desde el origen: $\vec{r}=\hat{x}\,\rho\cos\varphi+\hat{y}\,\rho\,\mathrm{sen}\,\varphi+\hat{z}\,z$
- diferencial de longitud genérico: $d\vec{l}=d\rho\,\hat{\rho}+\rho\,d\varphi\,\hat{\varphi}+dz\,\hat{z}$
- diferenciales de superficie principales: $d\vec{s}=\rho\,d\rho\,d\varphi\,\hat{z},\ d\rho\,dz\,\hat{\varphi},\ \rho\,d\varphi\,dz\,\hat{\rho}$
- diferencial de volumen: $dv=\rho\,d\varphi\,d\rho\,dz$
- vector genérico: $\vec{A}=A_\rho\,\hat{\rho}+A_\varphi\,\hat{\varphi}+A_z\,\hat{z}$
- vector unitario genérico: $\hat{r}=\dfrac{\hat{x}\,\rho\cos\varphi+\hat{y}\,\rho\,\mathrm{sen}\,\varphi+\hat{z}\,z}{\sqrt{\rho^2+z^2}}$

Los vectores unitarios principales son $\hat{\rho},\ \hat{\varphi},\ \hat{z}$ ($\hat{\rho}$ y $\hat{\varphi}$ dependen de la posición del punto considerado) con las relaciones: $\hat{\rho}\times\hat{\varphi}=\hat{z},\ \hat{z}\times\hat{\rho}=\hat{\varphi},\ \hat{\varphi}\times\hat{z}=\hat{\rho}$.

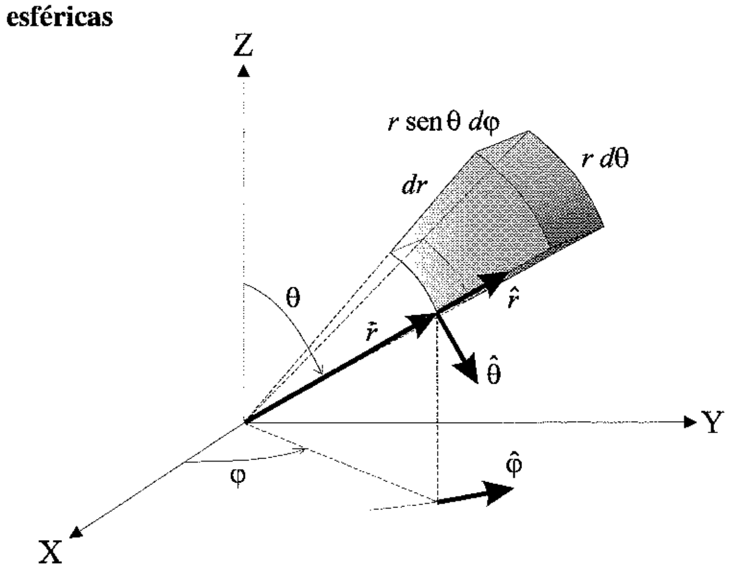
Cambio de sistema de coordenadas: de cilíndricas a cartesianas:

$$\hat{\rho}=\hat{x}\cos\varphi+\hat{y}\,\mathrm{sen}\,\varphi$$
$$\hat{\varphi}=-\hat{x}\,\mathrm{sen}\,\varphi+\hat{y}\cos\varphi$$
$$\hat{z}=\hat{z}$$

de cartesianas a cilíndricas:

$$\hat{x}=\hat{\rho}\cos\varphi-\hat{\varphi}\,\mathrm{sen}\,\varphi$$
$$\hat{y}=\hat{\rho}\,\mathrm{sen}\,\varphi+\hat{\varphi}\cos\varphi$$
$$\hat{z}=\hat{z}$$

Coordenadas esféricas



- radiovector desde el origen: $\vec{r}=\hat{x}r\,\mathrm{sen}\,\theta\cos\varphi+\hat{y}r\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi+\hat{z}r\cos\theta$
- diferencial de longitud genérico: $d\vec{l}=dr\,\hat{r}+r\,d\theta\,\hat{\theta}+r\,\mathrm{sen}\,\theta\,d\varphi\,\hat{\varphi}$
- diferenciales de superficie principales: $d\vec{s}=dr\,r\,d\theta\,\hat{\varphi},\ dr\,r\,\mathrm{sen}\,\theta\,d\varphi\,\hat{\theta},\ r^2\,\mathrm{sen}\,\theta\,d\theta\,d\varphi\,\hat{r}$
- diferencial de volumen: $dv=r^2\,\mathrm{sen}\,\theta\,dr\,d\theta\,d\varphi$
- vector genérico: $\vec{A}=A_r\,\hat{r}+A_\theta\,\hat{\theta}+A_\varphi\,\hat{\varphi}$

- vector unitario genérico: $\hat{r} = \dfrac{r\,\text{sen}\,\theta\cos\varphi\,\hat{x} + r\,\text{sen}\,\theta\,\text{sen}\,\varphi\,\hat{y} + r\cos\theta\,\hat{z}}{r}$

Los vectores unitarios principales son $\hat{r}$, $\hat{\theta}$, $\hat{\varphi}$ (todos ellos dependientes de la posición del punto en que se consideran) con las relaciones:
$\hat{r}\times\hat{\theta} = \hat{\varphi}$, $\hat{\varphi}\times\hat{r} = \hat{\theta}$, $\hat{\theta}\times\hat{\varphi} = \hat{r}$

Cambio de sistema de coordenadas: de esféricas a cartesianas

$$\hat{r} = \hat{x}\,\text{sen}\,\theta\cos\varphi + \hat{y}\,\text{sen}\,\theta\,\text{sen}\,\varphi + \hat{z}\,\cos\theta$$
$$\hat{\theta} = \hat{x}\,\cos\theta\cos\varphi + \hat{y}\,\cos\theta\,\text{sen}\,\varphi - \hat{z}\,\text{sen}\,\theta$$
$$\hat{\varphi} = -\hat{x}\,\text{sen}\,\varphi + \hat{y}\,\cos\varphi$$

de cartesianas a esféricas:

$$\hat{x} = \hat{r}\,\text{sen}\,\theta\cos\varphi + \hat{\theta}\,\cos\theta\cos\varphi - \hat{\varphi}\,\text{sen}\,\varphi$$
$$\hat{y} = \hat{r}\,\text{sen}\,\theta\,\text{sen}\,\varphi + \hat{\theta}\,\cos\theta\,\text{sen}\,\varphi + \hat{\varphi}\,\cos\varphi$$
$$\hat{z} = \hat{r}\,\cos\theta - \hat{\theta}\,\text{sen}\,\theta$$

Anexo B: Fórmulas de análisis vectorial

Operadores

Coordenadas rectangulares

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$
$$\nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$
$$\nabla \times A = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$$
$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$
$$\nabla^2 A = \nabla^2 A_x\hat{x} + \nabla^2 A_y\hat{y} + \nabla^2 A_z\hat{z} = \nabla(\nabla \cdot A) - \nabla \times (\nabla \times A)$$

Coordenadas cilíndricas

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\hat{\varphi} + \frac{\partial f}{\partial z}\hat{z}$$
$$\nabla \cdot A = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho A_\rho\right) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$
$$\nabla \times A = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\rho} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\varphi} + \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}\left(\rho A_\varphi\right) - \frac{\partial A_\rho}{\partial \phi}\right]\hat{z}$$
$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$