• Tarea 6

① Tablero de Butcher ¿Implícito?

$c_1 = 1/3, \; c_2 = 2/3$

- Sacamos el tablero de Butcher con el método de colocación:

$$q(\tau) = (\tau - 1/3)(\tau - 2/3)$$

$$q_1(\tau) = (\tau - 2/3) \longrightarrow q_1(c_1) = -1/3$$

$$q_2(\tau) = (\tau - 1/3) \longrightarrow q_2(c_2) = 1/3$$

$$a_{11} = \int_0^{1/3} \frac{q_1(\tau)}{q_1(c_1)} d\tau = -3\left[\frac{(\tau - 2/3)^2}{2}\right]_0^{1/3} = -3 \cdot \frac{1/9 - 4/9}{2} = \frac{1}{2}$$

$$a_{12} = \int_0^{1/3} \frac{q_2(\tau)}{q_2(c_2)} d\tau = 3\left[\frac{(\tau - 1/3)^2}{2}\right]_0^{1/3} = 3 \cdot \frac{-1/9}{2} = -\frac{1}{6}$$

$$a_{21} = \int_0^{2/3} \frac{q_1(\tau)}{q_1(c_1)} d\tau = -3\left[\frac{(\tau - 2/3)^2}{2}\right]_0^{2/3} = -3 \cdot \frac{-4/9}{2} = \frac{2}{3}$$

$$a_{22} = \int_0^{2/3} \frac{q_2(\tau)}{q_2(c_2)} d\tau = 3\left[\frac{(\tau - 1/3)^2}{2}\right]_0^{2/3} = 3 \cdot \frac{1/9 - 1/9}{2} = 0$$

$$b_1 = \int_0^1 \frac{q_1(\tau)}{q_1(c_1)} d\tau = -3\left[\frac{(\tau - 2/3)^2}{2}\right]_0^1 = -3 \cdot \frac{1/9 - 4/9}{2} = \frac{1}{2}$$

$$b_2 = \int_0^1 \frac{q_2(\tau)}{q_2(c_2)} d\tau = 3\left[\frac{(\tau - 1/3)^2}{2}\right]_0^1 = 3 \cdot \frac{4/9 - 1/9}{2} = \frac{1}{2}$$

• Tablero de Butcher:

$$\begin{array}{c|cc} 1/3 & 1/2 & -1/6 \\ 2/3 & 2/3 & 0 \\ \hline & 1/2 & 1/2 \end{array}$$

- Como $c_1 \neq 0 \Longrightarrow$ El método es <u>implícito</u>

② Función de estabilidad:

$$y_{n+1} = y_n + h b_1 f(t_n + c_1 h, \xi_1) + h b_2 f(t_n + c_2 h, \xi_2)$$

donde
$$\begin{cases} \xi_1 = y_n + h a_{11} f(t_n + c_1 h, \xi_1) + h a_{12} f(t_n + c_2 h, \xi_2) \\ \xi_2 = y_n + h a_{21} f(t_n + c_1 h, \xi_1) + h a_{22} f(t_n + c_2 h, \xi_2) \end{cases}$$

Tomamos $f(t,y) = \lambda y$:

$$\Longrightarrow \begin{cases} \xi_1 = y_n + \frac{h}{2}\lambda \xi_1 - \frac{h}{6}\lambda \xi_2 \\ \xi_2 = y_n + \frac{2}{3}h\lambda \xi_1 \end{cases}$$

$$\xrightarrow[\xi_2 \text{ en } 1^{a} \text{ igualdad}]{\text{Sustituimos}} \xi_1 = y_n + \frac{h}{2}\lambda \xi_1 - \frac{h}{6}\lambda y_n - \frac{h^2\lambda^2}{9}\xi_1$$

$$\Longrightarrow \xi_1 = \frac{1 - \frac{1}{6}h\lambda}{1 - \frac{1}{2}h\lambda + \frac{1}{9}h^2\lambda^2} y_n$$

$$\Longrightarrow \xi_2 = \frac{1 - \frac{1}{2}h\lambda + \cancel{\frac{1}{9}h^2\lambda^2} + \frac{2}{3}h\lambda - \cancel{\frac{1}{9}h^2\lambda^2}}{1 - \frac{1}{2}h\lambda + \frac{1}{9}h^2\lambda^2} y_n = \frac{1 + \frac{1}{6}h\lambda}{1 - \frac{1}{2}h\lambda + \frac{1}{9}h^2\lambda^2} y_n$$

Como $y_{n+1} = y_n + \frac{h\lambda}{2}\xi_1 + \frac{h\lambda}{2}\xi_2$

$$\Longrightarrow y_{n+1} = \frac{1 - \cancel{\frac{1}{2}h\lambda} + \frac{1}{9}h^2\lambda^2 + \frac{h\lambda}{2} - \cancel{\frac{1}{12}h^2\lambda^2} + \cancel{\frac{h\lambda}{2}} + \cancel{\frac{1}{12}h^2\lambda^2}}{1 - \frac{1}{2}h\lambda + \frac{1}{9}h^2\lambda^2} y_n$$

$$\Longrightarrow y_{n+1} = \frac{1 + \frac{h\lambda}{2} + \frac{h^2\lambda^2}{9}}{1 - \frac{h\lambda}{2} + \frac{h^2\lambda^2}{9}} y_n$$

$$\cancel{\Longrightarrow R(z) = \frac{1+z}{\;}}$$

$$\Longrightarrow R(z) = \frac{1 + \frac{z}{2} + \frac{z^2}{9}}{1 - \frac{z}{2} + \frac{z^2}{9}} \quad \text{con } z = h\lambda$$