

Derivadas parciales de primer orden de las siguientes funciones en un punto genérico. ¿Qué condiciones debe verificar este punto?

	$\frac{\partial f}{\partial x}(x, y)$	$\frac{\partial f}{\partial y}(x, y)$	Condiciones
a) $f(x, y) = \frac{2y}{(x+y)^2}$	$\frac{2y}{(x+y)^2}$	$\frac{-2x}{(x+y)^2}$	$x + y \neq 0$
b) $f(x, y) = \frac{10xy^2}{(x^2 + y^2)^2}$	$\frac{10xy^2}{(x^2 + y^2)^2}$	$\frac{-10x^2y}{(x^2 + y^2)^2}$	$x^2 + y^2 \neq 0$
c) $f(x, y) = 1$	1	1	$xy \neq 0$
d) $f(x, y) = \frac{y}{(x-y)^2} \sqrt{\frac{x-y}{x+y}}$	$-\frac{y}{(x-y)^2} \sqrt{\frac{x-y}{x+y}}$	$\frac{x}{(x-y)^2} \sqrt{\frac{x-y}{x+y}}$	$x - y \neq 0 \quad \frac{x+y}{x-y} > 0$
e) $f(x, y) = -2^{x+y} \ln 2 \frac{1}{(x+y)^2}$	$-2^{x+y} \ln 2 \frac{1}{(x+y)^2}$	$-2^{x+y} \ln 2 \frac{1}{(x+y)^2}$	$x + y \neq 0$
f) $f(x, y) = yx^{y-1} + y^x \ln y$	$yx^{y-1} + y^x \ln y$	$x^y \ln x + xy^{x-1}$	$x > 0 \quad y > 0$
g) $f(x, y) = \frac{-y/x^2}{1 + (y/x)^2} + \frac{1/y}{1 + (x/y)^2} = 0$	$\frac{-y/x^2}{1 + (y/x)^2} + \frac{1/y}{1 + (x/y)^2} = 0$	$\frac{1/x}{1 + (y/x)^2} + \frac{-x/y^2}{1 + (x/y)^2} = 0$	$x \neq 0 \quad y \neq 0$
h) $f(x, y) = \frac{e^x(-2x + x^2 - y^2)}{(x^2 - y^2)^2}$	$\frac{e^x(-2x + x^2 - y^2)}{(x^2 - y^2)^2}$	$\frac{2e^x y}{(x^2 - y^2)^2}$	$x^2 - y^2 \neq 0$
i) $f(x, y) = \frac{4xy^2}{(x^2 + y^2)^2}$	$\frac{4xy^2}{(x^2 + y^2)^2}$	$\frac{-4x^2y}{(x^2 + y^2)^2}$	$x^2 + y^2 \neq 0$

	$\frac{\partial f}{\partial x}(x, y, z)$	$\frac{\partial f}{\partial y}(x, y, z)$	$\frac{\partial f}{\partial z}(x, y, z)$	Condiciones
j) $f(x, y, z) = -\operatorname{tg}(x + y + z)$	$-\operatorname{tg}(x + y + z)$	$-\operatorname{tg}(x + y + z)$	$-\operatorname{tg}(x + y + z)$	$\cos(x + y + z) > 0 \rightarrow x + y + z < \frac{\pi}{2} + 2k\pi \quad k \in \mathbb{Z}$
k) $f(x, y, z) = -z \operatorname{sen}(xz + yz)$	$-z \operatorname{sen}(xz + yz)$	$-z \operatorname{sen}(xz + yz)$	$-(x + y) \operatorname{sen}(xz + yz)$	$\forall (x, y, z) \in \mathbb{R}^3$
l) $f(x, y, z) = -e^{-x} \operatorname{sen} y \cos z$	$-e^{-x} \operatorname{sen} y \cos z$	$e^{-x} \cos y \cos z$	$-e^{-x} \operatorname{sen} y \operatorname{sen} z$	$\forall (x, y, z) \in \mathbb{R}^3$

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
 LLAMA O ENVÍA WHATSAPP: 689 45 44 70
 ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
 CALL OR WHATSAPP: 689 45 44 70



www.cartagenas99.com no se hace responsable de la información contenida en el presente documento en virtud de la Ley de Servicios de la Sociedad de la Información y de Comercio Electrónico, de 11 de julio de 2002. Si la información contenida en el documento es ilícita o lesiona bienes o derechos de un tercero háganoslo saber y será retirada.

8 Calcular el vector gradiente de las siguientes funciones en un punto genérico y, si es posible, en el punto que se indica:

$$(a) f(x, y) = \frac{x + y}{xy} \quad P(1,1)$$

$$\begin{cases} D_1 f(x, y) = -\frac{y^2}{x^2 y^2} = -\frac{1}{x^2} \\ D_2 f(x, y) = -\frac{x^2}{x^2 y^2} = -\frac{1}{y^2} \end{cases} \rightarrow \nabla f(x, y) = \begin{pmatrix} -\frac{1}{x^2} \\ \frac{1}{y^2} \end{pmatrix} \xrightarrow{xy \neq 0} \nabla f(1,1) = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$(b) f(x, y) = \sqrt{x + y} \quad P(1,0)$$

$$\begin{cases} D_1 f(x, y) = \frac{1}{2\sqrt{x + y}} \\ D_2 f(x, y) = \frac{1}{2\sqrt{x + y}} \end{cases} \rightarrow \nabla f(x, y) = \begin{pmatrix} \frac{1}{2\sqrt{x + y}} \\ \frac{1}{2\sqrt{x + y}} \end{pmatrix} \xrightarrow{x+y > 0} \nabla f(1,0) = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

$$(c) f(x, y) = \frac{e^y}{x + y} \quad P(0,1)$$

$$\begin{cases} D_1 f(x, y) = \frac{-e^y}{(x + y)^2} \\ D_2 f(x, y) = \frac{e^y(x + y - 1)}{(x + y)^2} \end{cases} \rightarrow \nabla f(x, y) = \begin{pmatrix} \frac{-e^y}{(x + y)^2} \\ \frac{e^y(x + y - 1)}{(x + y)^2} \end{pmatrix} \xrightarrow{x+y \neq 0} \nabla f(0,1) = \begin{pmatrix} -e \\ 0 \end{pmatrix}$$

$$(d) f(x, y) = e^{-x} \text{sen}(x + y) \quad P(0, \pi)$$

$$\begin{cases} D_1 f(x, y) = -e^{-x} \text{sen}(x + y) + e^{-x} \cos(x + y) \\ D_2 f(x, y) = e^{-x} \cos(x + y) \end{cases} \rightarrow$$

$$\nabla f(x, y) = \begin{pmatrix} e^{-x}(\cos(x + y) - \text{sen}(x + y)) \\ e^{-x} \cos(x + y) \end{pmatrix} \quad \forall (x, y) \in \mathbb{R}^2 \rightarrow \nabla f(0, \pi) = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$



CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP:689 45 44 70

(f) $f(x, y, z) = 2^{\frac{z}{x+y}}$ $P(1,1,1)$

$$\begin{cases} D_1 f(x, y, z) = -2^{\frac{z}{x+y}} \text{Ln} 2 \frac{z}{(x+y)^2} \\ D_2 f(x, y, z) = -2^{\frac{z}{x+y}} \text{Ln} 2 \frac{z}{(x+y)^2} \\ D_3 f(x, y, z) = 2^{\frac{z}{x+y}} \text{Ln} 2 \left(\frac{1}{x+y} \right) \end{cases} \rightarrow \nabla f(x, y, z) = \begin{pmatrix} -\frac{2^{\frac{z}{x+y}} z \text{Ln} 2}{(x+y)^2} \\ -\frac{2^{\frac{z}{x+y}} z \text{Ln} 2}{(x+y)^2} \\ \frac{2^{\frac{z}{x+y}} \text{Ln} 2}{x+y} \end{pmatrix} \xrightarrow{x+y \neq 0} \nabla f(1,1,1) =$$

$$= \begin{pmatrix} \frac{-\sqrt{2} \text{Ln} 2}{4} \\ -\frac{\sqrt{2} \text{Ln} 2}{4} \\ \frac{\sqrt{2} \text{Ln} 2}{2} \end{pmatrix}$$

(g) $f(x, y, z) = \sqrt{xyz}$ $P(1,1,1)$

$$\begin{cases} D_1 f(x, y, z) = \frac{yz}{2\sqrt{xyz}} \\ D_2 f(x, y, z) = \frac{xz}{2\sqrt{xyz}} \\ D_3 f(x, y, z) = \frac{xy}{2\sqrt{xyz}} \end{cases} \rightarrow \nabla f(x, y, z) = \begin{pmatrix} \frac{yz}{2\sqrt{xyz}} \\ \frac{xz}{2\sqrt{xyz}} \\ \frac{xy}{2\sqrt{xyz}} \end{pmatrix} \xrightarrow{xyz > 0} \nabla f(1,1,1) = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$$

(h) $f(x, y, z, t) = (xt)^{yz}$ $P(1,1,1,1)$

$$\begin{cases} D_1 f(x, y, z, t) = yz(xt)^{yz-1}t \\ D_2 f(x, y, z, t) = (xt)^{yz} \text{Ln}(xt)z \\ D_3 f(x, y, z, t) = (xt)^{yz} \text{Ln}(xt)y \\ D_4 f(x, y, z, t) = yz(xt)^{yz-1}x \end{cases} \rightarrow \nabla f(x, y, z, t) = \begin{pmatrix} yz(xt)^{yz-1}t \\ (xt)^{yz} \text{Ln}(xt)z \\ (xt)^{yz} \text{Ln}(xt)y \\ yz(xt)^{yz-1}x \end{pmatrix} \xrightarrow{xt > 0} \nabla f(1,1,1,1) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$



CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP:689 45 44 70

Problemas resueltos derivadas de orden superior. Matriz hessiana. Matemáticas I.

9. Calcular el vector gradiente y la matriz hessiana de las siguientes funciones en un punto genérico y, si es posible, en el punto P que se indica:

$$(a) f(x, y) = \frac{x^3y - xy^3}{x^2 + y^2} \quad P(1,1)$$

Derivadas de primer orden:

$$\begin{cases} \frac{\partial f}{\partial x}(x, y) = \frac{(3x^2y - y^3)(x^2 + y^2) - 2x(x^3y - xy^3)}{(x^2 + y^2)^2} = \frac{x^4y + 4x^2y^3 - y^5}{(x^2 + y^2)^2} \\ \frac{\partial f}{\partial y}(x, y) = \frac{(x^3 - 3xy^2)(x^2 + y^2) - 2y(x^3y - xy^3)}{(x^2 + y^2)^2} = \frac{x^5 - 4x^3y^2 - xy^4}{(x^2 + y^2)^2} \end{cases} \xrightarrow{x^2 + y^2 \neq 0}$$

$$\nabla f(x, y) = \begin{pmatrix} \frac{x^4y + 4x^2y^3 - y^5}{(x^2 + y^2)^2} \\ \frac{x^5 - 4x^3y^2 - xy^4}{(x^2 + y^2)^2} \end{pmatrix} \rightarrow \nabla f(1,1) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Derivadas de segundo orden:

$$\boxed{\frac{\partial f}{\partial x}(x, y) = \frac{x^4y + 4x^2y^3 - y^5}{(x^2 + y^2)^2}} \rightarrow$$

$$\frac{\partial^2 f}{\partial x^2}(x, y) = \frac{(4x^3y + 8xy^3)(x^2 + y^2)^2 - 2(x^2 + y^2)2x(x^4y + 4x^2y^3 - y^5)}{(x^2 + y^2)^4} = \frac{-4x^3y^3 + 12xy^5}{(x^2 + y^2)^3}$$

$$\begin{aligned} \frac{\partial^2 f}{\partial y \partial x}(x, y) &= \frac{(x^4 + 12x^2y^2 - 5y^4)(x^2 + y^2)^2 - 2(x^2 + y^2)2y(x^4y + 4x^2y^3 - y^5)}{(x^2 + y^2)^4} \\ &= \frac{x^6 + 9x^4y^2 - 9x^2y^4 - y^6}{(x^2 + y^2)^3} \end{aligned}$$

$$\boxed{\frac{\partial f}{\partial y}(x, y) = \frac{x^5 - 4x^3y^2 - xy^4}{(x^2 + y^2)^2}} \rightarrow$$

$$\frac{\partial^2 f}{\partial x \partial y}(x, y) = \frac{(5x^4 - 12x^2y^2 - y^4)(x^2 + y^2)^2 - 2(x^2 + y^2)2x(x^5 - 4x^3y^2 - xy^4)}{(x^2 + y^2)^4} = \frac{x^6 + 9x^4y^2 - 9x^2y^4 - y^6}{(x^2 + y^2)^3}$$

$$\frac{\partial^2 f}{\partial y^2}(x, y) = \frac{(-8x^3y - 4xy^3)(x^2 + y^2)^2 - 2(x^2 + y^2)2y(x^5 - 4x^3y^2 - xy^4)}{(x^2 + y^2)^4} = \frac{-12x^5y + 4x^3y^3}{(x^2 + y^2)^3}$$

**CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70**

**ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP:689 45 44 70**

Cartagena99

Problemas resueltos derivadas de orden superior. Matriz hessiana. Matemáticas I.

(b) $f(x, y) = e^{xy}$ $P(1,0)$

Derivadas de primer orden:

$$\begin{cases} \frac{\partial f}{\partial x}(x, y) = y e^{xy} \\ \frac{\partial f}{\partial y}(x, y) = x e^{xy} \end{cases} \quad \forall (x, y) \in \mathbb{R}^2 \rightarrow \nabla f(x, y) = \begin{pmatrix} y e^{xy} \\ x e^{xy} \end{pmatrix} \rightarrow \nabla f(1,0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Derivadas de segundo orden:

$$\begin{cases} \frac{\partial f}{\partial x}(x, y) = y e^{xy} \rightarrow \begin{cases} \frac{\partial^2 f}{\partial x^2}(x, y) = y^2 e^{xy} \\ \frac{\partial^2 f}{\partial y \partial x}(x, y) = e^{xy}(xy + 1) \end{cases} \\ \frac{\partial f}{\partial y}(x, y) = x e^{xy} \rightarrow \begin{cases} \frac{\partial^2 f}{\partial x \partial y}(x, y) = e^{xy}(xy + 1) \\ \frac{\partial^2 f}{\partial y^2}(x, y) = x^2 e^{xy} \end{cases} \end{cases} \quad \forall (x, y) \in \mathbb{R}^2 \rightarrow$$

$$Hf(x, y) = \begin{pmatrix} y^2 e^{xy} & e^{xy}(xy + 1) \\ e^{xy}(xy + 1) & x^2 e^{xy} \end{pmatrix} \rightarrow Hf(1,0) = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

(c) $f(x, y) = x \ln(2y)$ $P = \left(0, \frac{e}{2}\right)$

Derivadas de primer orden:

$$\begin{cases} \frac{\partial f}{\partial x}(x, y) = \ln(2y) \\ \frac{\partial f}{\partial y}(x, y) = x \frac{2}{2y} \end{cases} \quad \text{con } y > 0 \rightarrow \nabla f(x, y) = \begin{pmatrix} \ln(2y) \\ \frac{x}{y} \end{pmatrix} \rightarrow \nabla f\left(0, \frac{e}{2}\right) = \begin{pmatrix} \ln e \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Derivadas de segundo orden:

$$\begin{cases} \frac{\partial f}{\partial x}(x, y) = \ln(2y) \rightarrow \begin{cases} \frac{\partial^2 f}{\partial x^2}(x, y) = 0 \\ \frac{\partial^2 f}{\partial y \partial x}(x, y) = \frac{2}{2y} = \frac{1}{y} \end{cases} \\ \frac{\partial f}{\partial y}(x, y) = \frac{x}{y} \rightarrow \begin{cases} \frac{\partial^2 f}{\partial x \partial y}(x, y) = \frac{1}{y} \\ \frac{\partial^2 f}{\partial y^2}(x, y) = -\frac{x}{y^2} \end{cases} \end{cases} \quad y > 0 \rightarrow Hf(x, y) = \begin{pmatrix} 0 & \frac{1}{y} \\ \frac{1}{y} & -\frac{x}{y^2} \end{pmatrix} \rightarrow Hf\left(0, \frac{e}{2}\right) = \begin{pmatrix} 0 & \frac{2}{e} \\ \frac{2}{e} & 0 \end{pmatrix}$$

**CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70**

**ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP:689 45 44 70**

Cartagena99

Problemas resueltos derivadas de orden superior. Matriz hessiana. Matemáticas I.

$$(d) f(x, y) = \cos(xy) + \operatorname{sen}(x^2y) \quad P\left(1, \frac{\pi}{2}\right)$$

Derivadas de primer orden:

$$\begin{cases} \frac{\partial f}{\partial x}(x, y) = -\operatorname{sen}(xy)y + \cos(x^2y)2xy \\ \frac{\partial f}{\partial y}(x, y) = -\operatorname{sen}(xy)x + \cos(x^2y)x^2 \end{cases} \xrightarrow{\forall(x,y) \in \mathbb{R}^2} \nabla f(x, y) = \begin{pmatrix} -\operatorname{sen}(xy)y + \cos(x^2y)2xy \\ -\operatorname{sen}(xy)x + \cos(x^2y)x^2 \end{pmatrix} \rightarrow$$

$$\nabla f\left(1, \frac{\pi}{2}\right) = \begin{pmatrix} \underbrace{-\operatorname{sen}\left(\frac{\pi}{2}\right)}_1 \frac{\pi}{2} + \underbrace{\cos\left(\frac{\pi}{2}\right)}_0 2 \frac{\pi}{2} \\ \underbrace{-\operatorname{sen}\left(\frac{\pi}{2}\right)}_1 + \underbrace{\cos\left(\frac{\pi}{2}\right)}_0 \end{pmatrix} = \begin{pmatrix} -\frac{\pi}{2} \\ -1 \end{pmatrix}$$

Derivadas de segundo orden:

$$\frac{\partial f}{\partial x}(x, y) = -\operatorname{sen}(xy)y + \cos(x^2y)2xy \rightarrow$$

$$\begin{cases} \frac{\partial^2 f}{\partial x^2}(x, y) = -\cos(xy)y^2 - \operatorname{sen}(x^2y)2xy \cdot 2xy + \cos(x^2y)2y \\ \frac{\partial^2 f}{\partial y \partial x}(x, y) = -\cos(xy)yx - \operatorname{sen}(xy) - \operatorname{sen}(x^2y)x^2 \cdot 2xy + \cos(x^2y)2x \end{cases}$$

$$\frac{\partial f}{\partial y}(x, y) = -\operatorname{sen}(xy)x + \cos(x^2y)x^2 \rightarrow$$

$$\begin{cases} \frac{\partial^2 f}{\partial x \partial y}(x, y) = -\cos(xy)yx - \operatorname{sen}(xy) - \operatorname{sen}(x^2y)2xyx^2 + 2x\cos(x^2y) \\ \frac{\partial^2 f}{\partial y^2}(x, y) = -x\cos(xy)x - \operatorname{sen}(x^2y)x^2 \cdot x^2 \end{cases}$$

$$\xrightarrow{\forall(x,y) \in \mathbb{R}^2} Hf(x, y) =$$

$$= \begin{pmatrix} -\cos(xy)y^2 - \operatorname{sen}(x^2y)4x^2y^2 + \cos(x^2y)2y & -\cos(xy)yx - \operatorname{sen}(xy) - \operatorname{sen}(x^2y)2x^3y + 2x\cos(x^2y) \\ -\cos(xy)yx - \operatorname{sen}(xy) - \operatorname{sen}(x^2y)2x^3y + 2x\cos(x^2y) & -x\cos(xy)x - \operatorname{sen}(x^2y)x^2 \cdot x^2 \end{pmatrix}$$

**CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70**

**ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP:689 45 44 70**

Cartagena99

Problemas resueltos derivadas de orden superior. Matriz hessiana. Matemáticas I.

$$(e) f(x, y, z) = \frac{x+z}{y-z} \quad P(0,1,-1)$$

Derivadas de primer orden:

$$\begin{cases} \frac{\partial f}{\partial x}(x, y, z) = \frac{1}{y-z} \\ \frac{\partial f}{\partial y}(x, y, z) = \frac{-(x+z)}{(y-z)^2} \\ \frac{\partial f}{\partial z}(x, y, z) = \frac{(y-z) - (-1)(x+z)}{(y-z)^2} = \frac{y+x}{(y-z)^2} \end{cases} \xrightarrow{y-z \neq 0} \nabla f(x, y, z) = \begin{pmatrix} \frac{1}{y-z} \\ \frac{-(x+z)}{(y-z)^2} \\ \frac{y+x}{(y-z)^2} \end{pmatrix} \rightarrow \nabla f(0,1,-1) = \begin{pmatrix} 1/2 \\ 1/4 \\ 1/4 \end{pmatrix}$$

Derivadas de segundo orden:

$$\begin{cases} \frac{\partial f}{\partial x}(x, y, z) = \frac{1}{y-z} \rightarrow \begin{cases} \frac{\partial^2 f}{\partial x^2}(x, y, z) = 0 \\ \frac{\partial^2 f}{\partial y \partial x}(x, y, z) = \frac{-1}{(y-z)^2} \\ \frac{\partial^2 f}{\partial z \partial x}(x, y, z) = \frac{1}{(y-z)^2} \end{cases} \\ \frac{\partial f}{\partial y}(x, y, z) = \frac{-(x+z)}{(y-z)^2} \rightarrow \begin{cases} \frac{\partial^2 f}{\partial x \partial y}(x, y, z) = \frac{-1}{(y-z)^2} \\ \frac{\partial^2 f}{\partial y^2}(x, y, z) = \frac{2(y-z)(x+z)}{(y-z)^4} = \frac{2(x+z)}{(y-z)^3} \\ \frac{\partial^2 f}{\partial z \partial y}(x, y, z) = -\frac{(y-z)^2 + 2(y-z)(x+z)}{(y-z)^4} = -\frac{(y-z) + 2(x+z)}{(y-z)^3} \end{cases} \\ \frac{\partial f}{\partial z}(x, y, z) = \frac{y+x}{(y-z)^2} \rightarrow \begin{cases} \frac{\partial^2 f}{\partial x \partial z}(x, y, z) = \frac{1}{(y-z)^2} \\ \frac{\partial^2 f}{\partial y \partial z}(x, y, z) = \frac{(y-z)^2 - 2(y-z)(y+x)}{(y-z)^4} = \frac{(y-z) - 2(y+x)}{(y-z)^3} \\ \frac{\partial^2 f}{\partial z^2}(x, y, z) = \frac{2(y-z)(y+x)}{(y-z)^4} = \frac{2(y+x)}{(y-z)^3} \end{cases} \end{cases}$$

$$Hf(x, y, z) = \begin{pmatrix} 0 & \frac{-1}{(y-z)^2} & \frac{1}{(y-z)^2} \\ \frac{-1}{(y-z)^2} & \frac{2(x+z)}{(y-z)^3} & \frac{-2x-y-z}{(y-z)^3} \\ 1 & \frac{-2x-y-z}{(y-z)^3} & \frac{2(y+x)}{(y-z)^3} \end{pmatrix} \xrightarrow{y-z \neq 0} Hf(0,1,-1) = \begin{pmatrix} 0 & \frac{-1}{4} & \frac{1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$$

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP:689 45 44 70

Cartagena99

Problemas resueltos derivadas de orden superior. Matriz hessiana. Matemáticas I.

$$(f) f(x, y, z) = \text{Ln}\left(\frac{x+y}{z}\right) \quad P(1,1,2)$$

Derivadas de primer orden:

$$\left\{ \begin{array}{l} \frac{\partial f}{\partial x}(x, y, z) = \frac{z}{x+y} \frac{1}{z} = \frac{1}{x+y} \\ \frac{\partial f}{\partial y}(x, y, z) = \frac{z}{x+y} \frac{1}{z} = \frac{1}{x+y} \\ \frac{\partial f}{\partial z}(x, y, z) = \frac{z}{x+y} \frac{-(x+y)}{z^2} = \frac{-1}{z} \end{array} \right. \xrightarrow{z \neq 0, \frac{x+y}{z} > 0} \nabla f(x, y, z) = \begin{pmatrix} \frac{1}{x+y} \\ \frac{1}{x+y} \\ \frac{-1}{z} \end{pmatrix} \rightarrow \nabla f(1,1,2) = \begin{pmatrix} 1/2 \\ 1/2 \\ -1/2 \end{pmatrix}$$

Derivadas de segundo orden:

$$\left\{ \begin{array}{l} \frac{\partial f}{\partial x}(x, y, z) = \frac{1}{x+y} \rightarrow \begin{cases} \frac{\partial^2 f}{\partial x^2}(x, y, z) = \frac{-1}{(x+y)^2} \\ \frac{\partial^2 f}{\partial y \partial x}(x, y, z) = \frac{-1}{(x+y)^2} \\ \frac{\partial^2 f}{\partial z \partial x}(x, y, z) = 0 \end{cases} \\ \frac{\partial f}{\partial y}(x, y, z) = \frac{1}{x+y} \rightarrow \begin{cases} \frac{\partial^2 f}{\partial x \partial y}(x, y, z) = \frac{-1}{(x+y)^2} \\ \frac{\partial^2 f}{\partial y^2}(x, y, z) = \frac{-1}{(x+y)^2} \\ \frac{\partial^2 f}{\partial z \partial y}(x, y, z) = 0 \end{cases} \\ \frac{\partial f}{\partial z}(x, y, z) = \frac{-1}{z} \rightarrow \begin{cases} \frac{\partial^2 f}{\partial x \partial z}(x, y, z) = 0 \\ \frac{\partial^2 f}{\partial y \partial z}(x, y, z) = 0 \\ \frac{\partial^2 f}{\partial z^2}(x, y, z) = \frac{1}{z^2} \end{cases} \end{array} \right. \xrightarrow{z \neq 0, \frac{x+y}{z} > 0}$$

$$Hf(x, y, z) = \begin{pmatrix} \frac{-1}{(x+y)^2} & \frac{-1}{(x+y)^2} & 0 \\ \frac{-1}{(x+y)^2} & \frac{-1}{(x+y)^2} & 0 \\ \frac{-1}{z^2} & 0 & \frac{1}{z^2} \end{pmatrix} \xrightarrow{\substack{x+y > 0 \\ z \neq 0}} Hf(1,1,2) = \begin{pmatrix} \frac{-1}{4} & \frac{-1}{4} & 0 \\ \frac{-1}{4} & \frac{-1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$$

**CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70**

**ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP:689 45 44 70**

