

Comparison tests for improper integrals (of functions of one variable)

- (1) Let $f, g : [a, \infty) \rightarrow \mathbb{R}$ be integrable functions on $[a, z]$ for all $z > a$.
- (a) If $0 \leq f \leq g$ on $[a, \infty)$ and the improper integral $\int_a^\infty g$ converges, then the improper integral $\int_a^\infty f$ converges.
 - (b) If $0 \leq f \leq g$ on $[a, \infty)$ and the improper integral $\int_a^\infty f$ diverges, then the improper integral $\int_a^\infty g$ diverges.
 - (c) If f and g are non-negative and continuous functions on $[a, \infty)$, $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$ and $\int_a^\infty g$ converges, then $\int_a^\infty f$ converges.
 - (d) If f and g are non-negative and continuous functions on $[a, \infty)$, $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \infty$ and $\int_a^\infty f$ diverges, then $\int_a^\infty g$ diverges.
 - (e) If f and g are nonnegative and continuous functions on $[a, \infty)$ and $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} \neq 0$ (and it is finite), then $\int_a^\infty f$ converges if and only if $\int_a^\infty g$ converges.
- (2) Let $f, g : (a, b] \rightarrow \mathbb{R}$ be integrable functions on $(z, b]$ for all $a < z < b$.
- (a) If $0 \leq f \leq g$ on $(a, b]$ and the improper integral $\int_a^b g$ converges, then the improper integral $\int_a^b f$ converges.
 - (b) If $0 \leq f \leq g$ on $(a, b]$ and the improper integral $\int_a^b f$ diverges, then the improper integral $\int_a^b g$ diverges.
 - (c) If f and g are non-negative functions on $(a, b]$, $\lim_{x \rightarrow a^+} \frac{f(x)}{g(x)} = 0$ and $\int_a^b g$ converges, then $\int_a^b f$ converges.
 - (d) If f and g are non-negative functions on $(a, b]$, $\lim_{x \rightarrow a^+} \frac{f(x)}{g(x)} = \infty$ and $\int_a^b f$ diverges, then $\int_a^b g$ diverges.
 - (e) If f and g are non-negative functions on $(a, b]$ and $\lim_{x \rightarrow a^+} \frac{f(x)}{g(x)} \neq 0$ (and it is finite), then $\int_a^b f$ converges if and only if $\int_a^b g$ converges.
- (3) (a) Let $f : [a, \infty) \rightarrow \mathbb{R}$ be an integrable function on $[a, z]$ for all $a < z$. If the improper

The logo for Cartagena99 features the word "Cartagena99" in a stylized, blue, serif font. The "99" is significantly larger and more prominent than the word "Cartagena". The logo is set against a light blue background with a white starburst effect behind the text.

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