PRÁCTICA 1 DE MÉTODOS MATEMÁTICOS I. 16 – 11 – 2020 (**1ºB**)

PROBLEMA 1. Indica los valores de $x \in \mathbb{R}$ para los que se cumple la desigualdad:

$$\frac{x-1}{(x+2)(x-3)} > 0$$

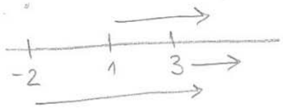
PROBLEMA 2. Demuestra por inducción que:

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots + \frac{n}{2^n} = 2 - \frac{n+2}{2^n}$$

PROB. 1 $\quad \dfrac{x-1}{(x+2)(x-3)} > 0$

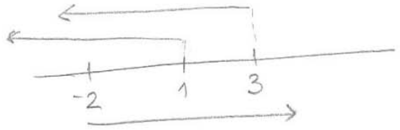
Casos: ① $x-1>0 \wedge (x+2)\cdot(x-3)>0 \Rightarrow \begin{cases} x>1 \wedge x+2>0 \wedge x-3>0 \\ \vee \\ x>1 \wedge x+2<0 \wedge x-3<0 \end{cases}$

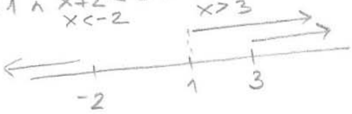
$x>1 \wedge x>-2 \wedge x>3$



$(3,\infty)$

② $x-1<0 \wedge (x+2)(x-3)<0 \Rightarrow \begin{cases} x<1 \wedge \underset{x>-2}{x+2>0} \wedge \underset{x<3}{x-3<0} \\ \vee \\ x<1 \wedge \underset{x<-2}{x+2<0} \wedge \underset{x>3}{x-3>0} \end{cases}$ $\quad (-2,1)$





$\emptyset$

Sol: $\forall x \in \left[(-2,1) \cup (3,\infty)\right]$

PROB. 2 $\quad \dfrac{1}{2} + \dfrac{2}{2^2} + \dfrac{3}{2^3} + \cdots + \dfrac{n}{2^n} = 2 - \dfrac{n+2}{2^n}$

Si $n=1$; $\dfrac{1}{2} \stackrel{?}{=} 2 - \dfrac{3}{2}$ ✓

Si $n=2$; $\underbrace{\dfrac{1}{2} + \dfrac{2}{2^2}}_{1} = \underbrace{2 - \dfrac{4}{4}}_{1}$ ✓

⋮

Si se cumple que: $\dfrac{1}{2} + \dfrac{2}{2^2} + \cdots + \dfrac{n-1}{2^{n-1}} = 2 - \dfrac{n+1}{2^{n-1}}$

¿ $\dfrac{1}{2} + \dfrac{2}{2^n} + \cdots + \dfrac{n-1}{2^{n-1}} + \dfrac{n}{2^n} = 2 - \dfrac{n+2}{2^n}$?

$$\left(\frac{1}{2} + \frac{2}{2^n} + \cdots + \frac{n-1}{2^{n-1}}\right) + \frac{n}{2^n} = \left(2 - \frac{n+1}{2^{n-1}}\right) + \frac{n}{2^n} = \frac{2^n - n - 1}{2^{n-1}} + \frac{n}{2^n} =$$

$$= \frac{2^{n+1} - 2n - 2 + n}{2^n} = \frac{2^{n+1} - n - 2}{2^n} = 2 - \frac{n+2}{2^n} \quad //$$