2º BG. TECNOLÓGICO. EXA 4. Temas 4, 5 y 6.

1º. $f(x)=\frac{(x-1)^2}{x-2}$. $Df: \mathbb{R}-\{2\}$. $f'(x)=\frac{(x-1)(x-3)}{(x-2)^2}=\frac{x^2-4x+3}{(x-2)^2}$.

$f''(x)=\frac{2}{(x-2)^2}$.

<u>asíntotas:</u>

. $\lim_{x\to 2^+}\frac{(x-1)^2}{x-2}=+\infty$; $\lim_{x\to 2^-}\frac{(x-1)^2}{x-2}=-\infty$; Asínt. vertical: $x=2$

. $\lim_{x\to\infty}\frac{(x-1)^2}{x-2}=\infty$. No hay asíntota horizontal

. $m=\lim_{x\to\infty}\frac{(x-1)^2}{x^2-2x}=\lim_{x\to\infty}\frac{x^2}{x^2}=1$; $n=\lim_{x\to\infty}\frac{(x-1)^2}{x-2}-x=\lim_{x\to\infty}\frac{1}{x-2}=0$.

asíntota oblicua $y=x$ tanto si $x\to+\infty$ como si $x\to-\infty$

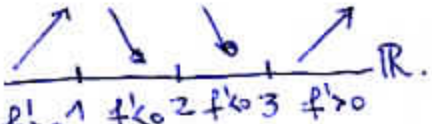
<u>Extremos:</u>

$f'(x)=0$. $x^2-4x+3=0$; $x=\frac{4\pm\sqrt{16-12}}{2}=\frac{4\pm 2}{2}=\begin{cases}3\\1\end{cases}$

$f''(3)=\frac{2}{(3-2)^2}=2>0$. En $x=3$ hay mín. relativo y vale $f(3)=4$.

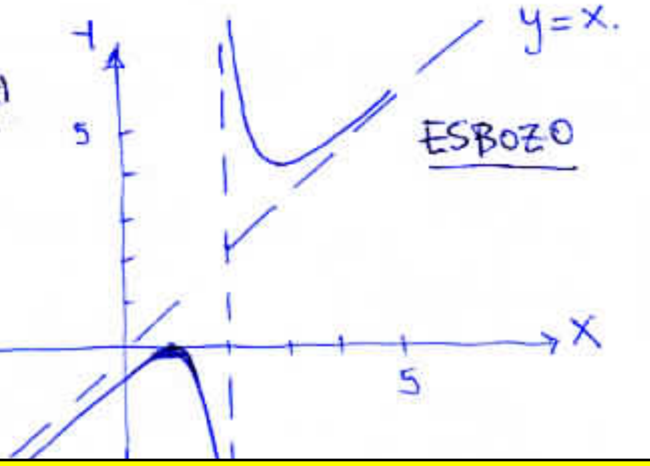
$f''(1)=\frac{2}{(1-2)^2}=-2<0$ En $x=1$ hay máx. relativo y vale $f(1)=0$

<u>Estudio signo de $f'(x)$</u>



Creciente: $(-\infty,1)\cup(3,+\infty)$

Decreciente: $(1,3)$



$f'(x)=(x-10)$; $f(x)=\frac{(x-10)}{10}$.

3° $\displaystyle\int \frac{x^2}{\sqrt[3]{1+2x}}\,dx = *$ Cambio de variable: $1+2x = t^3$;

$x = \dfrac{t^3-1}{2}$; $dx = \dfrac{3t^2}{2}\,dt$.

$$* = \int \frac{\left[\frac{t^3-1}{2}\right]^2}{t}\cdot\frac{3t^2}{2}\,dt = \frac{3}{8}\int (t^3-1)^2\cdot t\,dt = \frac{3}{8}\int (t^7-2t^4+t)\,dt$$

$$= \frac{3}{8}\left[\frac{t^8}{8} - \frac{2t^5}{5} + \frac{t^2}{2}\right] + cte = \frac{3}{8}\left[\frac{\sqrt[3]{(1+2x)^8}}{8} - \frac{2\sqrt[3]{(1+2x)^5}}{5} + \frac{\sqrt[3]{(1+2x)^2}}{2}\right] + cte$$

4° $$\int \frac{3x-7}{x^2+x+5}\,dx = \frac{3}{2}\int \frac{\frac{2}{3}(x-7)}{x^2+x+5}\,dx = \frac{3}{2}\int \frac{2x-\frac{14}{3}+1-1}{x^2+x+5}\,dx =$$

$$= \frac{3}{2}\left[\int \frac{2x+1}{x^2+x+5}\,dx + \int \frac{-14/3-1}{x^2+x+5}\,dx\right] = \frac{3}{2}\ln|x^2+x+5| +$$

$$+ \frac{3}{2}\cdot\left(-\frac{17}{3}\right)\int \frac{1}{x^2+x+5}\,dx \underset{*}{=} \frac{3}{2}\ln|x^2+x+5| - \frac{17}{\cancel{2}}\frac{\cancel{2}\sqrt{19}}{19}\,\mathrm{arctg}\left[\frac{2x+1}{\sqrt{19}}\right] + cte$$

$$\int \frac{1}{x^2+x+5}\,dx = \int \frac{1}{(x+\frac{1}{2})^2+5-\frac{1}{4}}\,dx = \int \frac{dx}{\frac{19}{4}+(x+\frac{1}{2})^2} =$$

$$= \int \frac{dx}{\frac{19}{4}\left[1+\frac{(x+1/2)^2}{19/4}\right]} = \frac{4}{19}\int \frac{dx}{1+\left[\frac{2(x+1/2)}{\sqrt{19}}\right]^2} = \frac{4}{19}\int \frac{dx}{1+\left[\frac{2x+1}{\sqrt{19}}\right]^2} =$$

$$= \frac{4}{19}\frac{\sqrt{19}}{2}\int \frac{2/\sqrt{19}}{1+\left[\frac{2x+1}{\sqrt{19}}\right]^2}\,dx = \frac{2\sqrt{19}}{19}\,\mathrm{arctg}\left[\frac{2x+1}{\sqrt{19}}\right] + cte.$$

5° $y = **$, $y = \sqrt{2x}$

$$A = \int_0^2 \overbrace{\sqrt{2x} - (-\sqrt{2x})}^{2\sqrt{2}x^{1/2}}\,dx + \int_2^4 \left[\sqrt{2x} - (1+\sqrt{2})x + 4 + 2\sqrt{2}\right]dx$$

$y = -(1+\sqrt{2})x + 4 + 2\sqrt{2}$

$$- (1+\sqrt{2})8 + (1+\sqrt{2})2 + 8 + 4\sqrt{2} =$$

$$= \ldots = \boxed{\frac{14+10\sqrt{2}}{3}\,u^2}$$