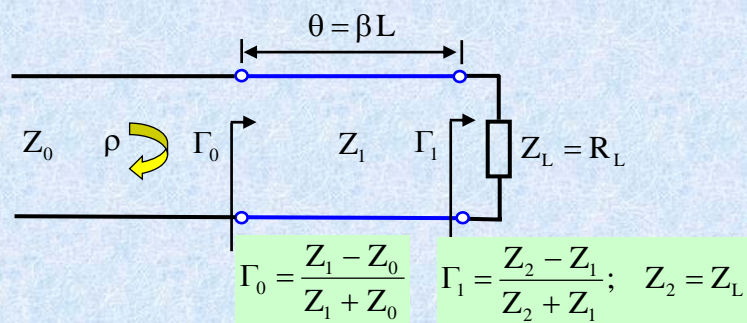


LÍNEAS DE TRANSMISIÓN

ADAPTACIÓN DE IMPEDANCIAS EN BANDA ANCHA

Banda ancha

Teoría de pequeñas reflexiones

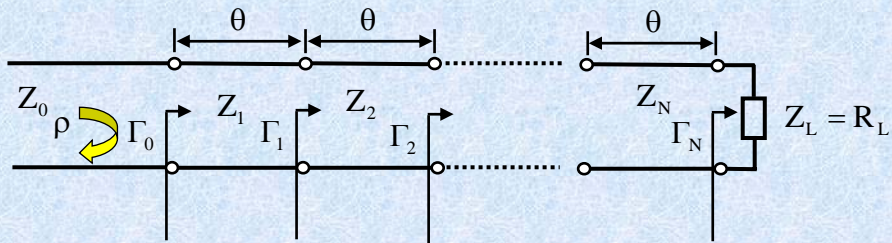


$$\rho = \frac{\Gamma_0 + \Gamma_1 e^{-j2\theta}}{1 + \Gamma_0 \Gamma_1 e^{-j2\theta}}$$

Si $|\Gamma_0|, |\Gamma_1| \ll 1$ \longrightarrow $\rho \approx \Gamma_0 + \Gamma_1 e^{-j2\theta}$

Banda ancha

Teoría de pequeñas reflexiones



$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$\rho(\theta) = \Gamma_0 + \Gamma_1 e^{-j2\theta} + \Gamma_2 e^{-j4\theta} + \dots + \Gamma_N e^{-j2N\theta}$$

Banda ancha

Si la red es simétrica:



$$\Gamma_0 = \Gamma_N, \Gamma_1 = \Gamma_{N-1}, \dots$$

$$\rho(\theta) = \begin{cases} e^{-jN\theta} \left[\Gamma_0 (e^{jN\theta} + e^{-jN\theta}) + \Gamma_1 (e^{j(N-2)\theta} + e^{-j(N-2)\theta}) + \dots + \Gamma_{(N-1)/2} (e^{j\theta} + e^{-j\theta}) \right] & \text{N impar} \\ e^{-jN\theta} \left[\Gamma_0 (e^{jN\theta} + e^{-jN\theta}) + \Gamma_1 (e^{j(N-2)\theta} + e^{-j(N-2)\theta}) + \dots + \Gamma_{N/2} \right] & \text{N par} \end{cases}$$

$$\rho(\theta) = \begin{cases} 2e^{-jN\theta} \left[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + \dots + \Gamma_n \cos(N-2n)\theta + \dots + \frac{1}{2} \Gamma_{(N-1)/2} \cos \theta \right] & \text{N impar} \\ 2e^{-jN\theta} \left[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + \dots + \Gamma_n \cos(N-2n)\theta + \dots + \frac{1}{2} \Gamma_{N/2} \right] & \text{N par} \end{cases}$$

$$\text{Dado } \rho(\theta) \Rightarrow Z_1, Z_2, \dots, Z_n$$

Banda ancha

Transformador binomial

$$\rho(\theta) = A(1 + e^{-j2\theta})^N = A \sum_{n=0}^N C_n^N e^{-j2n\theta} = \Gamma_0 + \Gamma_1 e^{-j2\theta} + \Gamma_2 e^{-j4\theta} + \dots + \Gamma_N e^{-j2N\theta}$$

$$\Gamma_n = A C_n^N$$

$$\rho(\theta) = A e^{-jN\theta} (e^{j\theta} + e^{-j\theta})^N = 2^N A e^{-jN\theta} \cos^N \theta$$

Máxima respuesta plana:

$$\left. \frac{d^n |\rho(\theta)|}{d\theta^n} \right|_{\theta=\frac{\pi}{2}, 1=\frac{\lambda}{4}} = 0; \quad n = 1, 2, \dots, N-1$$

$$\rho(0) = 2^N A = \frac{Z_L - Z_0}{Z_L + Z_0}$$



$$A = 2^{-N} \frac{Z_L - Z_0}{Z_L + Z_0}$$

Banda ancha

Transformador binomial

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \approx \frac{1}{2} \ln \frac{Z_{n+1}}{Z_n}$$

$$\ln \frac{Z_{n+1}}{Z_n} \approx 2\Gamma_n = 2A C_n^N = 2 \cdot 2^{-N} \frac{Z_L - Z_0}{Z_L + Z_0} C_n^N \approx 2^{-N} C_n^N \ln \frac{Z_L}{Z_0}$$

Ancho de banda:

$$|\rho_m| = 2^N |A| \cos^N \theta_m$$

$$\theta_m = \arccos \left[\frac{1}{2} \left(\frac{|\rho_m|}{|A|} \right)^{1/N} \right]$$

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi} \arccos \left[\frac{1}{2} \left(\frac{|\rho_m|}{|A|} \right)^{1/N} \right]$$

$N \uparrow, \Delta f \uparrow$

Banda ancha

Transformador binomial Ejemplo:

$$Z_L = 50 \Omega, Z_0 = 100 \Omega, N = 3, \rho_m = 0,05$$

$$A = 2^{-N} \frac{Z_L - Z_0}{Z_L + Z_0} \approx \frac{1}{2^{N+1}} \ln \frac{Z_L}{Z_0} = -0,0433$$

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \arccos \left[\frac{1}{2} \left(\frac{|\rho_m|}{|A|} \right)^{1/N} \right] = 0,70 \rightarrow 70 \%$$

$$C_0^3 = \binom{3}{0} = \frac{3!}{3!0!} = 1$$

$$C_1^3 = \binom{3}{1} = \frac{3!}{2!1!} = 3$$

$$C_2^3 = \binom{3}{2} = \frac{3!}{1!2!} = 3$$

Banda ancha

Transformador binomial Ejemplo:

$$Z_L = 50 \Omega, Z_0 = 100 \Omega, N = 3, \rho_m = 0,05$$

$$n = 0 \rightarrow \ln Z_1 = \ln Z_0 + 2^{-3} C_0^3 \ln \frac{Z_L}{Z_0} = 4,518$$

$$Z_1 = 91,7 \Omega$$

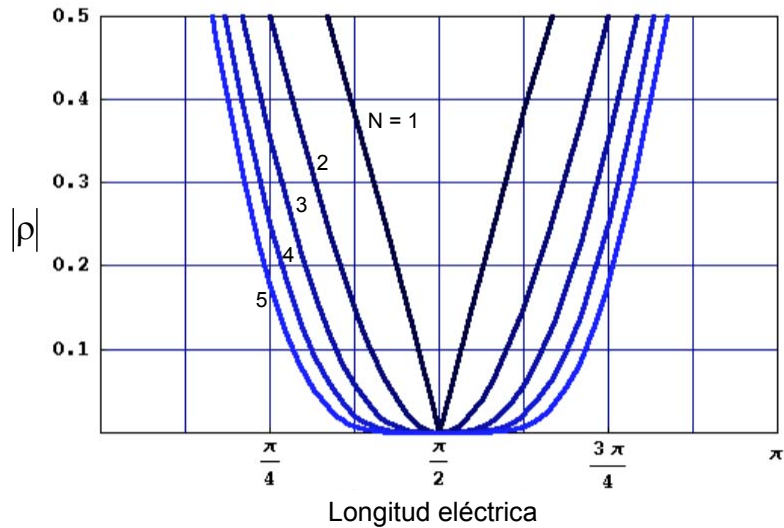
$$n = 1 \rightarrow \ln Z_2 = \ln Z_1 + 2^{-3} C_1^3 \ln \frac{Z_L}{Z_0} = 4,26$$

$$Z_2 = 70,7 \Omega$$

$$n = 2 \rightarrow \ln Z_3 = \ln Z_2 + 2^{-3} C_2^3 \ln \frac{Z_L}{Z_0} = 4,00$$

$$Z_3 = 54,5 \Omega$$

Transformador Binomial



Banda ancha

Transformador binomial

| Z_L/Z_0 | $N = 2$ | | $N = 3$ | | | $N = 4$ | | | |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| | Z_1/Z_0 | Z_2/Z_0 | Z_1/Z_0 | Z_2/Z_0 | Z_3/Z_0 | Z_1/Z_0 | Z_2/Z_0 | Z_3/Z_0 | Z_4/Z_0 |
| 1.0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 1.5 | 1.1067 | 1.3554 | 1.0520 | 1.2247 | 1.4259 | 1.0257 | 1.1351 | 1.3215 | 1.4624 |
| 2.0 | 1.1892 | 1.6818 | 1.0907 | 1.4142 | 1.8337 | 1.0444 | 1.2421 | 1.6102 | 1.9150 |
| 3.0 | 1.3161 | 2.2795 | 1.1479 | 1.7321 | 2.6135 | 1.0718 | 1.4105 | 2.1269 | 2.7990 |
| 4.0 | 1.4142 | 2.8285 | 1.1907 | 2.0000 | 3.3594 | 1.0919 | 1.5442 | 2.5903 | 3.6633 |
| 6.0 | 1.5651 | 3.8336 | 1.2544 | 2.4495 | 4.7832 | 1.1215 | 1.7553 | 3.4182 | 5.3500 |
| 8.0 | 1.6818 | 4.7568 | 1.3022 | 2.8284 | 6.1434 | 1.1436 | 1.9232 | 4.1597 | 6.9955 |
| 10.0 | 1.7783 | 5.6233 | 1.3409 | 3.1623 | 7.4577 | 1.1613 | 2.0651 | 4.8424 | 8.6110 |

| Z_L/Z_0 | $N = 5$ | | | | | $N = 6$ | | | | | |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| | Z_1/Z_0 | Z_2/Z_0 | Z_3/Z_0 | Z_4/Z_0 | Z_5/Z_0 | Z_1/Z_0 | Z_2/Z_0 | Z_3/Z_0 | Z_4/Z_0 | Z_5/Z_0 | Z_6/Z_0 |
| 1.0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 1.5 | 1.0128 | 1.0790 | 1.2247 | 1.3902 | 1.4810 | 1.0064 | 1.0454 | 1.1496 | 1.3048 | 1.4349 | 1.4905 |
| 2.0 | 1.0220 | 1.1391 | 1.4142 | 1.7558 | 1.9569 | 1.0110 | 1.0790 | 1.2693 | 1.5757 | 1.8536 | 1.9782 |
| 3.0 | 1.0354 | 1.2300 | 1.7321 | 2.4390 | 2.8974 | 1.0176 | 1.1288 | 1.4599 | 2.0549 | 2.6577 | 2.9481 |
| 4.0 | 1.0452 | 1.2995 | 2.0000 | 3.0781 | 3.8270 | 1.0225 | 1.1661 | 1.6129 | 2.4800 | 3.4302 | 3.9120 |
| 6.0 | 1.0596 | 1.4055 | 2.4495 | 4.2689 | 5.6625 | 1.0296 | 1.2219 | 1.8573 | 3.2305 | 4.9104 | 5.8275 |
| 8.0 | 1.0703 | 1.4870 | 2.8284 | 5.3800 | 7.4745 | 1.0349 | 1.2640 | 2.0539 | 3.8950 | 6.3291 | 7.7302 |
| 10.0 | 1.0789 | 1.5541 | 3.1623 | 6.4346 | 9.2687 | 1.0392 | 1.2982 | 2.2215 | 4.5015 | 7.7030 | 9.6228 |

Banda ancha

Transformador Chebyshev

$$\rho(\theta) = 2e^{-jN\theta} [\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + \dots + \Gamma_n \cos(N-2n)\theta + \dots]$$

$$\rho(\theta) = A e^{-jN\theta} T_N(\sec\theta_m \cos\theta) \Rightarrow \Gamma_n$$

Polinomios de Chebyshev:

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

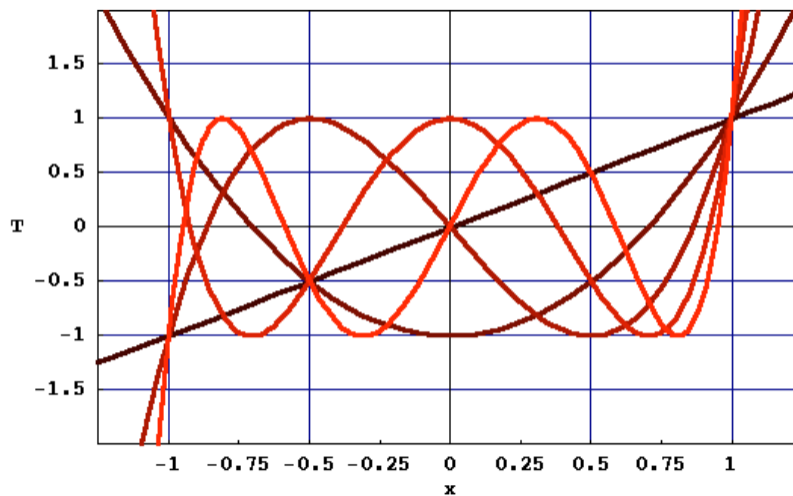
$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

$$x = \frac{\cos\theta}{\cos\theta_m}$$

Polinomios de Chebyshev



Banda ancha

Transformador Chebyshev

$$\rho(0) = A T_N(\sec \theta_m) = \frac{Z_L - Z_0}{Z_L + Z_0} \longrightarrow A = \frac{Z_L - Z_0}{Z_L + Z_0} \frac{1}{T_N(\sec \theta_m)} = \rho_m$$

$$T_N(\sec \theta_m) = \frac{1}{\rho_m} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| \longrightarrow \theta_m \longrightarrow \frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi}$$

- Dado $\rho_m \rightarrow$ máximo Δf
- Dado $\Delta f \rightarrow$ mínimo ρ_m
- Mayor ancho de banda que con el transformador binomial
- Rizado en la banda de paso

Banda ancha

Transformador Chebyshev Ejemplo:

$$Z_L = 100 \Omega, Z_0 = 50 \Omega, N = 3, \rho_m = 0,05$$

$$\begin{aligned} \rho(\theta) &= 2e^{-j3\theta} (\Gamma_0 \cos 3\theta + \Gamma_1 \cos \theta) = A e^{-j3\theta} T_3(\sec \theta_m \cos \theta) \\ &= A e^{-j3\theta} (4 \sec^3 \theta_m \cos^3 \theta - 3 \sec \theta_m \cos \theta) \\ &= A e^{-j3\theta} [\sec^3 \theta_m (\cos 3\theta + 3 \cos \theta) - 3 \sec \theta_m \cos \theta] \end{aligned}$$

$$A = \frac{Z_L - Z_0}{Z_L + Z_0} \frac{1}{T_3(\sec \theta_m)} = \rho_m \longrightarrow \sec \theta_m$$

$$\theta_m = 44,7^\circ \longrightarrow \frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} = 101 \%$$

Banda ancha

Transformador Chebyshev Ejemplo:

$$Z_L = 100 \Omega, Z_0 = 50 \Omega, N = 3, \rho_m = 0,05$$

$$2\Gamma_0 = A \sec^3 \theta_m \rightarrow \Gamma_0 = 0,0698 = \Gamma_3$$

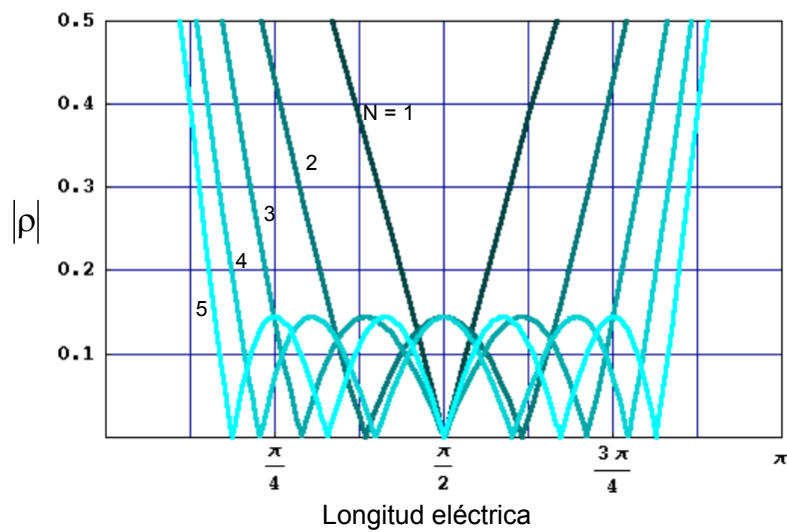
$$2\Gamma_1 = A(3\sec^3 \theta_m - 3\sec \theta_m) \rightarrow \Gamma_1 = 0,1037 = \Gamma_2$$

$$\Gamma_0 = \frac{Z_1 - Z_0}{Z_1 + Z_0} \rightarrow Z_1 = 57,5 \Omega$$

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1} \rightarrow Z_2 = 70,7 \Omega$$

$$\Gamma_2 = \frac{Z_3 - Z_2}{Z_3 + Z_2} \rightarrow Z_3 = 87 \Omega$$

Transformador Chebyshev



Banda ancha Transformador Chebyshev

| Z_L/Z_0 | $N = 2$ | | | | $N = 3$ | | | | | |
|-----------|-------------------|-----------|-------------------|-----------|-------------------|-----------|-----------|-------------------|-----------|-----------|
| | $\Gamma_m = 0.05$ | | $\Gamma_m = 0.20$ | | $\Gamma_m = 0.05$ | | | $\Gamma_m = 0.20$ | | |
| | Z_1/Z_0 | Z_2/Z_0 | Z_1/Z_0 | Z_2/Z_0 | Z_1/Z_0 | Z_2/Z_0 | Z_3/Z_0 | Z_1/Z_0 | Z_2/Z_0 | Z_3/Z_0 |
| 1.0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 1.5 | 1.1347 | 1.3219 | 1.2247 | 1.2247 | 1.1029 | 1.2247 | 1.3601 | 1.2247 | 1.2247 | 1.2247 |
| 2.0 | 1.2193 | 1.6402 | 1.3161 | 1.5197 | 1.1475 | 1.4142 | 1.7429 | 1.2855 | 1.4142 | 1.5558 |
| 3.0 | 1.3494 | 2.2232 | 1.4565 | 2.0598 | 1.2171 | 1.7321 | 2.4649 | 1.3743 | 1.7321 | 2.1829 |
| 4.0 | 1.4500 | 2.7585 | 1.5651 | 2.5558 | 1.2662 | 2.0000 | 3.1591 | 1.4333 | 2.0000 | 2.7908 |
| 6.0 | 1.6047 | 3.7389 | 1.7321 | 3.4641 | 1.3383 | 2.4495 | 4.4833 | 1.5193 | 2.4495 | 3.9492 |
| 8.0 | 1.7244 | 4.6393 | 1.8612 | 4.2983 | 1.3944 | 2.8284 | 5.7372 | 1.5766 | 2.8284 | 5.0742 |
| 10.0 | 1.8233 | 5.4845 | 1.9680 | 5.0813 | 1.4385 | 3.1623 | 6.9517 | 1.6415 | 3.1623 | 6.0920 |

| Z_L/Z_0 | $N = 4$ | | | | | | | |
|-----------|-------------------|-----------|-----------|-----------|-------------------|-----------|-----------|-----------|
| | $\Gamma_m = 0.05$ | | | | $\Gamma_m = 0.20$ | | | |
| | Z_1/Z_0 | Z_2/Z_0 | Z_3/Z_0 | Z_4/Z_0 | Z_1/Z_0 | Z_2/Z_0 | Z_3/Z_0 | Z_4/Z_0 |
| 1.0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 1.5 | 1.0892 | 1.1742 | 1.2775 | 1.3772 | 1.2247 | 1.2247 | 1.2247 | 1.2247 |
| 2.0 | 1.1201 | 1.2979 | 1.5409 | 1.7855 | 1.2727 | 1.3634 | 1.4669 | 1.5715 |
| 3.0 | 1.1586 | 1.4876 | 2.0167 | 2.5893 | 1.4879 | 1.5819 | 1.8965 | 2.0163 |
| 4.0 | 1.1906 | 1.6414 | 2.4369 | 3.3597 | 1.3692 | 1.7490 | 2.2870 | 2.9214 |
| 6.0 | 1.2290 | 1.8773 | 3.1961 | 4.8820 | 1.4415 | 2.0231 | 2.9657 | 4.1623 |
| 8.0 | 1.2583 | 2.0657 | 3.8728 | 6.3578 | 1.4914 | 2.2428 | 3.5670 | 5.3641 |
| 10.0 | 1.2832 | 2.2268 | 4.4907 | 7.7930 | 1.5163 | 2.4210 | 4.1305 | 6.5950 |

Comparación Binomial-Chebyshev Transformador de tres secciones

