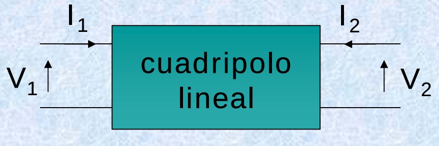
TEMA 2

PARÁMETROS S

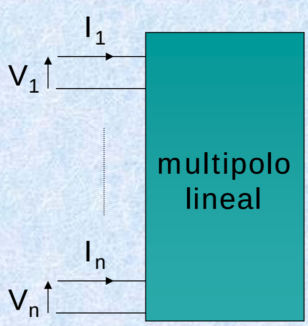
PARÁMETROS S

- Cuadripolos

Representación



Definidos con n^2 párametros

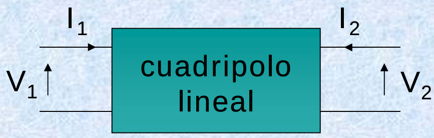


PARÁMETROS S

- Parámetros clásicos
  - Relacionan V,I entrada con V,I salida

Ej. Parámetros "z"

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$



Ej. Parámetros "h"

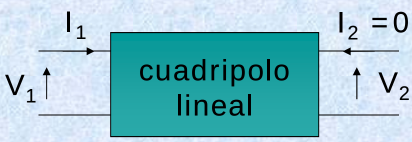
$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

PARÁMETROS S

- Medida de parámetros clásicos

Ej. Medida de z_{11}

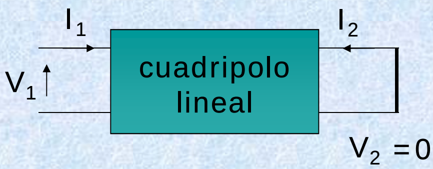
$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = z_{11}$$



C. abierto

Ej. Medida de h_{21}

$$I_2 = h_{21}I_1 + h_{22}V_2$$
$$\left.\frac{I_2}{I_1}\right|_{V_2=0} = h_{21}$$



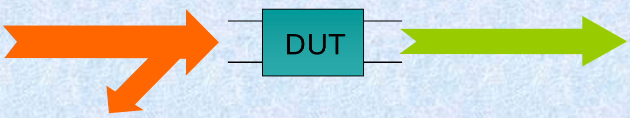
C. circuito

PARÁMETROS S

- Necesidad de parámetros "S"
  Inconvenientes de "z", "y", "g" y "h" en A.F.
  - Dificultad para conseguir C.C. y C.A.
  - Dificultad para medir tensiones y corrientes
  - Los dipositivos activos pueden presentar inestabilidad con C.C. o C.A. o autodestruirse

PARÁMETROS S

- Magnitudes típicas en microondas



REFLEXIÓN	TRANSMISIÓN
Coeficiente de reflexión	Ganancia
Pérdidas de retorno	Atenuación
R.O.E.	P. Inserción

→ Interesan parámetros que usen estas magnitudes

PARÁMETROS S

- Definición de ondas de potencia

Onda entrante al dispositivo $a_i(z_i, f) = \dfrac{V_i^+(z_i, f)}{\sqrt{\mathrm{Re}(Z_i)}}$

Onda saliente del dispositivo $b_i(z_i, f) = \dfrac{V_i^-(z_i, f)}{\sqrt{\mathrm{Re}(Z_i)}}$

➡ **Variables para definir "S"**

Dependen de

$z_i \rightarrow$ posición

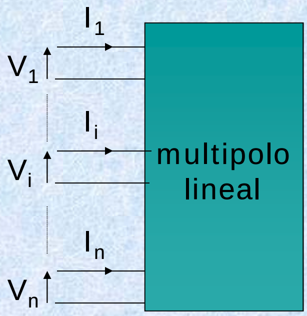
$f \rightarrow$ frecuencia

PARÁMETROS S

$$V_i = V_i^+ + V_i^- \qquad I_i = \frac{1}{Z_i}\left(V_i^+ - V_i^-\right)$$

$$a_i = \frac{V_i^+}{\sqrt{\mathrm{Re}(Z_i)}} \qquad a_i = \frac{V_i + I_i Z_i}{2\sqrt{\mathrm{Re}(Z_i)}}$$

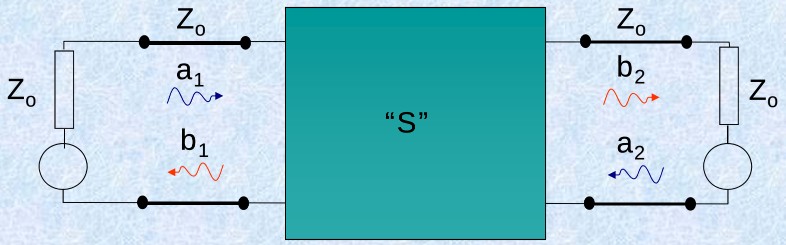
$$b_i = \frac{V_i^-}{\sqrt{\mathrm{Re}(Z_i)}} \qquad b_i = \frac{V_i - I_i Z_i^*}{2\sqrt{\mathrm{Re}(Z_i)}}$$



Z_i Impedancia de referencia

PARÁMETROS S

En la práctica $Z_{01}=Z_{02}=....=Z_{0n}= Z_0=50\ \Omega$
Los parámetros S dependen de la impedancia de referencia



$$a_i = \frac{V_i^+}{\sqrt{\mathrm{Re}(Z_i)}} \qquad b_i = \frac{V_i^-}{\sqrt{\mathrm{Re}(Z_i)}}$$

PARÁMETROS S

- Ecuaciones lineales del cuadripolo
  - Variables independientes: a_i
  - Variables dependientes: b_i
  - Comportamiento definido por n^2 parámetros

Dos puertas:

$$b_1 = s_{11}a_1 + s_{12}a_2$$

$$b_2 = s_{21}a_1 + s_{22}a_2$$

¡¡ Es necesario conocer Z_{ref} !!

PARÁMETROS S

- Ecuaciones lineales de un multipolo
  - Redes de n puertas
  - Microondas: habitual 3 y 4 puertas

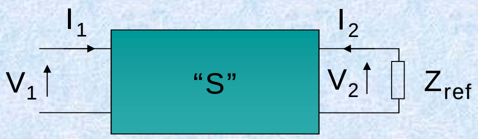
Para n puertas:

$$
\begin{aligned}
b_1 &= s_{11}a_1 + s_{12}a_2 + \cdots + s_{1n}a_n \\
b_2 &= s_{21}a_1 + s_{22}a_2 + \cdots + s_{2n}a_n \\
&\vdots \\
b_n &= s_{n1}a_1 + s_{n2}a_2 + \cdots + s_{nn}a_n
\end{aligned}
$$

PARÁMETROS S

- Determinación de los parámetros
  - Para determinar s_{11} y s_{21} $a_2=0$
  - Para determinar s_{12} y s_{22} $a_1=0$

Ej. Cargando la puerta 2 con Z_{ref}:



$b_1 = s_{11}a_1 + s_{12}a_2$ $b_2 = s_{21}a_1 + s_{22}a_2$

PARÁMETROS S

- Significado de los parámetros

$\left.\frac{b_1}{a_1}\right|_{a_2=0} = s_{11}$ Coeficiente de reflexión puerto 1 con $Z_2 = Z_{ref}$

$\left.\frac{b_2}{a_2}\right|_{a_1=0} = s_{22}$ Coeficiente de reflexión puerto 2 con $Z_1 = Z_{ref}$

$\left.\frac{b_1}{a_2}\right|_{a_1=0} = s_{12}$ Ganancia compleja (puerto 2 a 1) con $Z_1 = Z_{ref}$

$\left.\frac{b_2}{a_1}\right|_{a_2=0} = s_{21}$ Ganancia compleja (puerto 1 a 2) con $Z_2 = Z_{ref}$

PARÁMETROS S

- Interpretación práctica
  - Dispositivo caracterizado con "S" (Z_{ref}=50 Ω)
  - Conectado a sistema de 50 Ω
  - "S" describe directamente su comportamiento:

ρ entrada

τ puerta 2-1

$$\begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}$$

τ puerta 1-2

ρ salida

PARÁMETROS S

- Propiedades de la matrices Z e Y
  - Si la red no tiene pérdidas: $\mathrm{Re}\{Z\} = 0; \ \mathrm{Re}\{Y\} = 0$
  - Si la red es recíproca o simétrica: $Z = Z^T; \ Y = Y^T$
- Propiedades de la matriz de parámetros S
  - Si la red es pasiva: $|s_{ii}| \leq 1, \ |s_{ij}| \leq 1$
  - Si la red no tiene pérdidas: $S \cdot S^+ = I \Rightarrow S^+ \triangleq S^{*T} = S^{T^*}$
  - Si la red tiene pérdidas: $I - S \cdot S^+$ es definida positiva
  - Si la red es recíproca o simétrica: $S = S^T$

PARÁMETROS S

- Cambios entre matrices
  - Definiciones:

$$[U] = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \qquad [Z_0] = \begin{bmatrix} Z_{01} & 0 & \cdots & 0 \\ 0 & Z_{02} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Z_{0n} \end{bmatrix}$$

  - Por tanto

$$[Z_0]^{-1} = \begin{bmatrix} Z_{01}^{-1} & 0 & \cdots & 0 \\ 0 & Z_{02}^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Z_{0n}^{-1} \end{bmatrix} \qquad [Z_0]^{-\frac{1}{2}} = \begin{bmatrix} Z_{01}^{-\frac{1}{2}} & 0 & \cdots & 0 \\ 0 & Z_{02}^{-\frac{1}{2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Z_{0n}^{-\frac{1}{2}} \end{bmatrix}$$

PARÁMETROS S

- Cambios entre matrices
  - Entonces:

$$\begin{cases} V_i = V_i^+ + V_i^- \\ I_i = \dfrac{V_i^+ - V_i^-}{Z_{0i}} \end{cases} \Rightarrow \begin{cases} \{V^+\} = 0.5\left(\{V\} + [Z_0]\cdot\{I\}\right) \\ \{V^-\} = 0.5\left(\{V\} - [Z_0]\cdot\{I\}\right) \end{cases}$$

$$\Rightarrow \begin{cases} \{a\} = 0.5[Z_0]^{-\frac{1}{2}}\cdot\left(\{V\} + [Z_0]\cdot\{I\}\right) \\ \{b\} = 0.5[Z_0]^{-\frac{1}{2}}\cdot\left(\{V\} - [Z_0]\cdot\{I\}\right) \end{cases}$$

$$\Rightarrow \begin{cases} \{V\} = [Z_0]^{\frac{1}{2}}\cdot\left(\{a\} + \{b\}\right) \\ \{I\} = [Z_0]^{-\frac{1}{2}}\cdot\left(\{a\} - \{b\}\right) \end{cases}$$

PARÁMETROS S

- Cambios entre matrices
  - De Z a S:

$$\{b\} = [S]\cdot\{a\} \Rightarrow$$

$$[Z_0]^{-\frac{1}{2}}\cdot\left(\{V\} - [Z_0]\cdot\{I\}\right) = [S]\cdot[Z_0]^{-\frac{1}{2}}\cdot\left(\{V\} + [Z_0]\cdot\{I\}\right) \Rightarrow$$

$$[Z_0]^{-\frac{1}{2}}\cdot\left([Z] - [Z_0]\right)\cdot\{I\} = [S]\cdot[Z_0]^{-\frac{1}{2}}\cdot\left([Z] + [Z_0]\right)\cdot\{I\} \Rightarrow$$

$$[Z_0]^{-\frac{1}{2}}\cdot\left([Z] - [Z_0]\right) = [S]\cdot[Z_0]^{-\frac{1}{2}}\cdot\left([Z] + [Z_0]\right) \Rightarrow$$

$$[S] = [Z_0]^{-\frac{1}{2}}\cdot\left([Z] - [Z_0]\right)\cdot\left([Z] + [Z_0]\right)^{-1}\cdot[Z_0]^{\frac{1}{2}}$$

PARÁMETROS S

- Cambios entre matrices
  - De S a Z:

$$\{V\} = [Z]\cdot\{I\} \Rightarrow$$

$$[Z_0]^{\frac{1}{2}}\cdot([U]+[S])\cdot\{a\} = [Z]\cdot[Z_0]^{\frac{1}{2}}\cdot([U]-[S])\cdot\{a\}$$

$$[Z_0]^{\frac{1}{2}}\cdot([U]+[S]) = [Z]\cdot[Z_0]^{\frac{1}{2}}\cdot([U]-[S])$$

$$Z = [Z_0]^{\frac{1}{2}}\cdot([U]+[S])\cdot([U]-[S])^{-1}\cdot[Z_0]^{\frac{1}{2}}$$

PARÁMETROS S

- Cambios entre matrices
  - De S a Y:

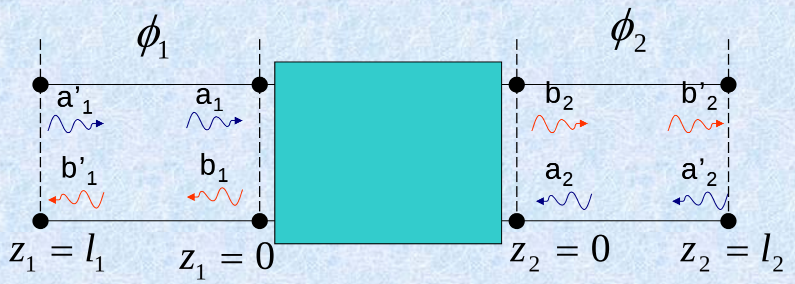
$$Y = [Z_0]^{-\frac{1}{2}}\cdot([U]-[S])\cdot([U]+[S])^{-1}\cdot[Z_0]^{-\frac{1}{2}}$$

  - De Y a S

$$S = [Z_0]^{\frac{1}{2}}\cdot\left([Z_0]^{-1}-[Y]\right)\cdot\left([Z_0]^{-1}+[Y]\right)^{-1}\cdot[Z_0]^{-\frac{1}{2}}$$

PARÁMETROS S

- Cambio del plano de referencia



$$a_1 = a_1' \, e^{-j\phi_1} \qquad a_2 = a_2' \, e^{-j\phi_2}$$

$$b_1 = b_1' \, e^{j\phi_1} \qquad b_2 = b_2' \, e^{j\phi_2}$$

PARÁMETROS S

$$a_i = a'_i \, e^{-j\phi_i} \Rightarrow \{a\} = [D]\cdot\{a'\}$$

$$b_i = b'_i \, e^{j\phi_i} \Rightarrow \{b\} = [D]^*\cdot\{b'\}$$

$$[D] = \begin{bmatrix} e^{-j\phi_1} & 0 & \cdots & 0 \\ 0 & e^{-j\phi_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-j\phi_n} \end{bmatrix}$$

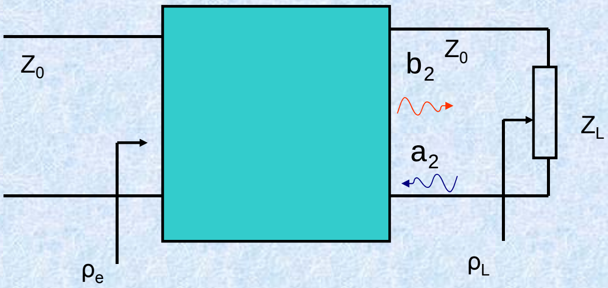
$$\{b\} = [S]\cdot\{a\} \Rightarrow [D]^*\cdot\{b'\} = [S]\cdot[D]\cdot\{a'\} \Rightarrow$$

$$\{b'\} = [D]\cdot[S]\cdot[D]\cdot\{a'\} \Rightarrow [S'] = [D]\cdot[S]\cdot[D]$$

PARÁMETROS S

Cálculo del coeficiente de reflexión a la entrada

$$\rho_L = \frac{a_2}{b_2}$$



$$a_2 = \rho_L b_2$$

$$b_1 = s_{21} a_1 + s_{22} a_2$$

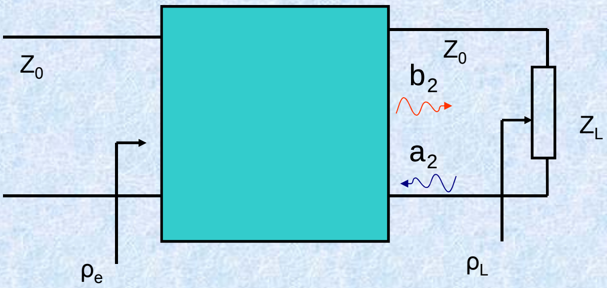
$$b_2 = s_{21} a_1 + s_{22} \rho_L b_2$$

$$b_1 = s_{11} a_1 + s_{12} a_2$$

$$b_2 = \frac{s_{21} a_1}{1 - \rho_L s_{22}} \Rightarrow a_2 = \frac{s_{21} a_1 \rho_L}{1 - \rho_L s_{22}}$$

$$b_1 = s_{11} a_1 + \frac{s_{12} s_{21} \rho_L a_1}{1 - \rho_L s_{22}}$$

PARÁMETROS S

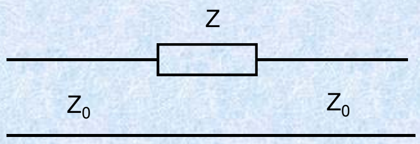


$$\frac{b_1}{a_1} = \rho_e = s_{11} + \frac{s_{12} s_{21} \rho_L}{1 - \rho_L s_{22}}$$

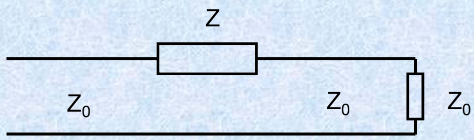
$$\frac{b_2}{a_2} = \rho_s = s_{22} + \frac{s_{12} s_{21} \rho_g}{1 - \rho_g s_{11}}$$

PARÁMETROS S

- Parámetros S de una impedancia serie

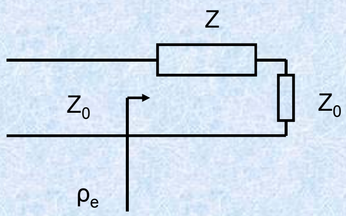


- Para calcular s_{11} y s_{21}, terminamos el puerto 2 con Z_0



PARÁMETROS S

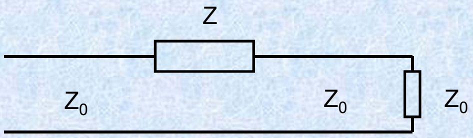
- Calculamos el coeficiente de reflexión en el puerto 1



$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \rho_e = \frac{Z + Z_0 - Z_0}{Z + Z_0 + Z_0} = \frac{Z}{Z + 2Z_0}$$

PARÁMETROS S

- Para calcular s_{21}, aplicamos que la corriente total en el puerto 1 es igual a la corriente total en el puerto 2



$$s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

$$I_1^+ + I_1^- = I_2^+ + I_2^- \quad ; \qquad \text{siendo} \quad I_2^+ = 0$$

$$\frac{V_1^+ - V_1^-}{Z_0} = \frac{V_2^-}{Z_0}$$

PARÁMETROS S

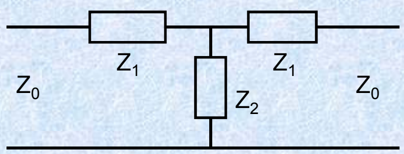
$$a_1 - b_1 = b_2 \quad \Rightarrow \quad 1 - \frac{b_1}{a_1} = \frac{b_2}{a_1} \quad \Rightarrow \quad 1 - s_{11} = s_{21}$$

$$s_{21} = 1 - s_{11} = 1 - \frac{Z}{Z + 2Z_0} = \frac{2Z_0}{Z + 2Z_0}$$

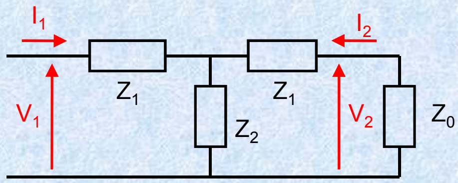
$$s_{11} = s_{22} = \frac{Z}{Z + 2Z_0} \qquad s_{21} = s_{12} = \frac{2Z_0}{Z + 2Z_0}$$

PARÁMETROS S

- Parámetros S de una impedancia serie

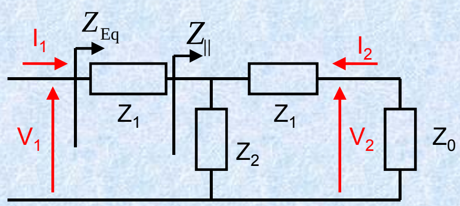


- Para calcular s_{11} y s_{21}, terminamos el puerto 2 con Z_0



PARÁMETROS S

- Definimos



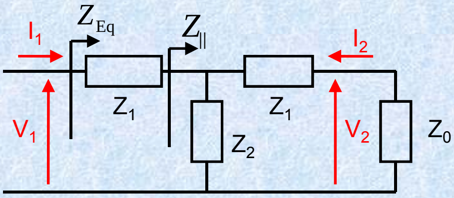
$$Z_{\|} = Z_2 \,\|\, (Z_1 + Z_0) = \frac{Z_2(Z_1 + Z_0)}{Z_0 + Z_1 + Z_2}$$

$$Z_{\mathrm{Eq}} = Z_{\|} + Z_1 = \frac{Z_0Z_1 + Z_0Z_2 + 2Z_1Z_2 + Z_0^{\,2}}{Z_0 + Z_1 + Z_2}$$

- Entonces

$$s_{11=} \frac{Z_{\mathrm{Eq}} - Z_0}{Z_{\mathrm{Eq}} + Z_0} = \frac{2Z_1Z_2 + Z_1^{\,2} - Z_0^{\,2}}{(Z_0 + Z_1)(Z_0 + Z_1 + 2Z_2)}$$

PARÁMETROS S

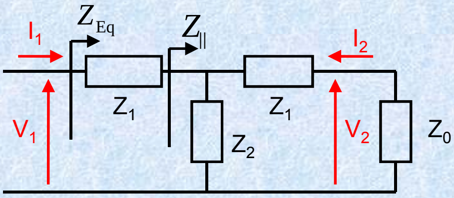


$$s_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_1^-=0} = \frac{V_2}{0.5\left(V_1 + Z_0 I_1\right)}$$

$$I_1 Z_{\|} = -I_2\left(Z_0 + Z_1\right) \qquad V_2 = -I_2 Z_0 = \frac{Z_0 Z_{\|}}{\left(Z_0 + Z_1\right)} I_1 \qquad V_1 = Z_{\mathrm{Eq}} I_1$$

$$s_{21} = \frac{2\dfrac{Z_0 Z_{\|}}{\left(Z_0 + Z_1\right)} I_1}{\left(Z_{\mathrm{Eq}} + Z_0\right) I_1} = \frac{2 Z_0 Z_{\|}}{\left(Z_0 + Z_1\right)\left(Z_{\mathrm{Eq}} + Z_0\right)} = \frac{2 Z_0 Z_2}{\left(Z_0 + Z_1\right)\left(Z_0 + Z_1 + 2Z_2\right)}$$

PARÁMETROS S



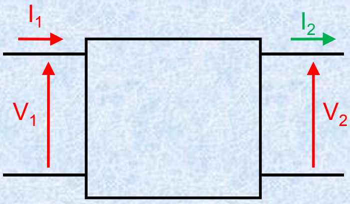
$$s_{21} = s_{21}; \quad s_{11} = s_{22}$$

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_1 + Z_2 \end{bmatrix}$$

$$[S] = [Z_0]^{-\frac{1}{2}} \cdot \left([Z] - [Z_0]\right) \cdot \left([Z] + [Z_0]\right)^{-1} \cdot [Z_0]^{\frac{1}{2}}$$

PARÁMETROS S

- Parámetros ABCD



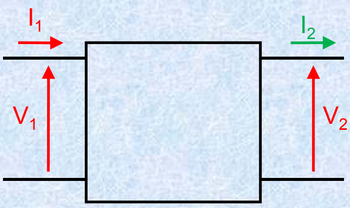
$$\begin{Bmatrix} V_1 \\ I_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} V_2 \\ I_2 \end{Bmatrix}$$

- Parámetros ABCD de una línea de transmisión sin pérdidas

$$\begin{Bmatrix} V(z) \\ I(z) \end{Bmatrix} = \begin{bmatrix} \cos(\beta z) & jZ_0\, sen(\beta z) \\ \dfrac{j}{Z_0} sen(\beta z) & \cos(\beta z) \end{bmatrix} \begin{Bmatrix} V_L \\ I_L \end{Bmatrix}$$

PARÁMETROS S

- Parámetros ABCD



$$\begin{Bmatrix} V_1 \\ I_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} V_2 \\ I_2 \end{Bmatrix}$$

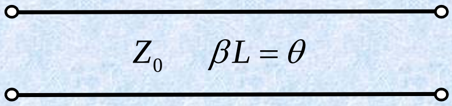
- Identificando

$$\begin{Bmatrix} V_1 \\ I_1 \end{Bmatrix} = \begin{bmatrix} \cos(\beta z) & jZ_0\, sen(\beta z) \\ \dfrac{j}{Z_0} sen(\beta z) & \cos(\beta z) \end{bmatrix} \begin{Bmatrix} V_2 \\ I_2 \end{Bmatrix}$$

PARÁMETROS S

- Parámetros S de una línea de transmisión sin pérdidas

$$\begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}$$



$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = 0 = s_{22} \qquad s_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} = \frac{V_1^-}{V_2^+} = e^{-j\theta} = \frac{V_1}{V_2} = s_{21}$$

$$S = \begin{bmatrix} 0 & e^{-j\theta} \\ e^{-j\theta} & 0 \end{bmatrix}$$