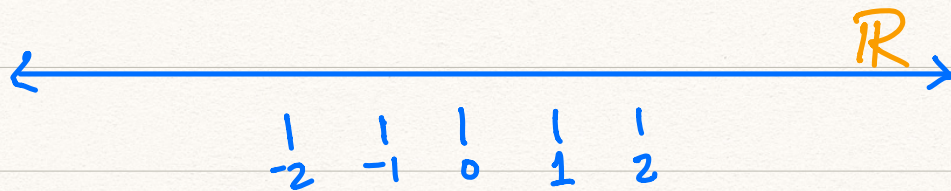


Intuitively, the completeness of $\mathbb{R}$ can be understood as follows:

If $\mathbb{R}$ is represented as a straight line that extends indefinitely in both directions, the completeness of $\mathbb{R}$ corresponds to the continuous nature of the line (solid line, no gaps).



Bounds

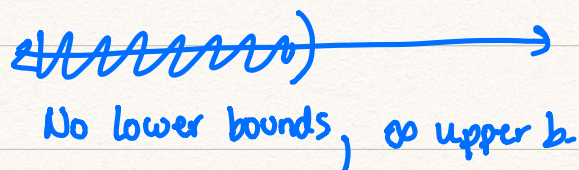
- ▶ A set $S \subseteq \mathbb{R}$ is bounded above if there exists some $H \in \mathbb{R}$ such that $x \leq H$ for every $x \in S$. If H exists, it is called an upper bound of S .

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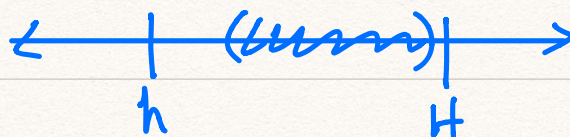
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- A set $S \subseteq \mathbb{R}$ is bounded below if there exists some $h \in \mathbb{R}$ such that $h \leq x$ for every $x \in S$. If h exists, it is called a lower bound of S .



- A set which is both bounded below and above is just said to be bounded.



Examples:

(i) $\{1, 2, 3\}$ Bounded

Bounded above : upper bounds 3, 10, 1000

bounded below : lower bounds 1, -10, -1000

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Bounded below: $1, 0, -12, -1M$.

(iii) $\{x: x > 0\}$ ~~$\leftarrow$~~

Not bounded above because for eve H in the set,
 $H+1 > H$ and $H+1$ is also in the set

Bounded below: $0, -1, -15, \dots$

Continuum property

- ▶ Every non-empty set of real numbers which is bounded above has a smallest upper bound. (Supremum $B = \sup S$)
- ▶ Every non-empty set of real numbers which is bounded below has a greatest lower bound. (Infimum $b = \inf S$)

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Examples:

(i) $\{1, 2, 3\}$

Supremum: 3

Infimum: 1

(ii) $\{x : 1 \leq x < 2\}$

Infimum: 1

Supremum: 2

(iii) $\{x : x > 0\}$

Unbounded above $\Rightarrow$ no supremum

Infimum: 0

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b

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Maximum and minimum

If a non-empty set S is bounded above and $B = \sup S \in S$, then B is called the maximum of S .

If a non-empty set S is bounded below and $b = \inf S \in S$, then b is called the minimum of S .

The logo for Cartagena99, featuring the text "Cartagena99" in a stylized font with a blue and orange background.

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Examples:

(i) $\{1, 2, 3\}$ Not an interval

Maximum? yes 3

Minimum? yes 1

(ii) $\{x : 1 \leq x < 2\}$ Interval

Maximum? No

Minimum? yes 1

(iii) $\{x : x > 0\}$ Interval

Max? No, not bounded above.

Min? No

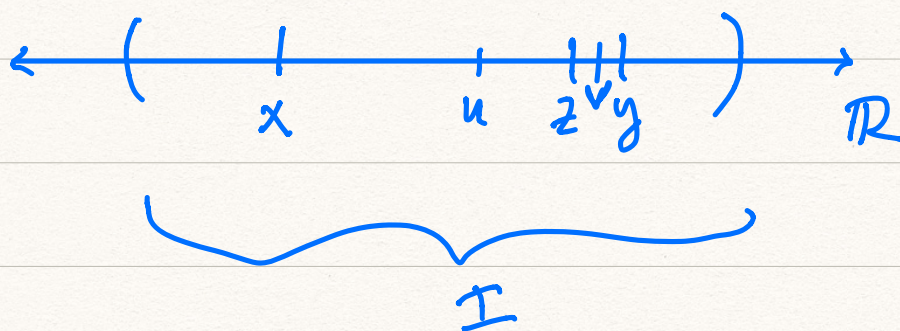
The logo for Cartagena99 features the text 'Cartagena99' in a stylized, bold font. The '99' is significantly larger and more prominent than the word 'Cartagena'. The text is dark green or black, set against a light blue background with a subtle gradient and a faint orange shape at the bottom.

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Intervals

An interval I is a set of real numbers with the property that, if $x \in I$ and $y \in I$ and $x \leq z \leq y$, then $z \in I$.



Notation:

► Bounded intervals:

1. $(a, b) = \{x : a < x < b\}$

2. $[a, b] = \{x : a \leq x \leq b\}$

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► Unbounded intervals:

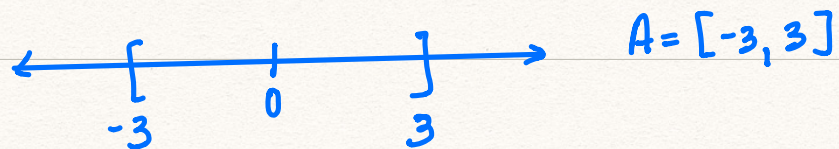
$$5. (a, \infty) = \{x : x > a\}$$

$$6. [a, \infty) = \{x : x \geq a\}$$

$$7. (-\infty, b) = \{x : x < b\}$$

$$8. (-\infty, b] = \{x : x \leq b\}$$

Example: Identify the set $A = \{x : |x| \leq 3\}$.



Case I: if $x > 0$ then $|x| = x$

$$\text{so } |x| \leq 3 \Rightarrow x \leq 3$$

Case II: if $x \leq 0$, then $|x| = -x$

$$\text{so } |x| \leq 3 \Rightarrow -x \leq 3 \text{ or } -3 \leq x$$

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Example: Identify the set

$$B = \{x: (x-1)(x-2)(x-3) < 0\} = (-\infty, 1) \cup (2, 3)$$

	$-\infty$	1	2	3	∞
	$\leftarrow$				$\rightarrow$
$x-1$	-	+	+	+	
$x-2$	-	-	+	+	
$x-3$	-	-	-	+	
$(x-1)(x-2)(x-3)$	-	+	-	+	

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