Aerodynamics (AER)

LESSON 1:
INTRO TO AERODYNAMICS
- GENERAL EQUATIONS -

José I. Rojas
josep.ignasi.rojas@upc.edu
Building C3, Office 120

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LESSON 1: INTRO – GENERAL EQUATIONS

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INTRO – GENERAL EQUATIONS

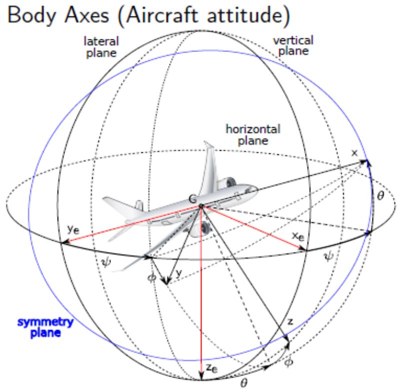
INTRODUCTION

- Global objective → to calculate aerodynamic forces on aircraft in horizontal motion in steady atmosphere ($V_w = 0$)
- Wind reference frame:
  - bonded to aircraft (origin in CG or elsewhere in symmetry plane)
  - x_w-axis → along direction of incident flow
  - z_w-axis → contained in vertical plane, opposite to gravity
  - y_w-axis → chosen such that we have a dextrogyre reference frame
- Forces: drag $F_x = D$, lift $F_z = L$, & lateral force $F_y = Q$
- Hypotheses: constant mass, rigid body, symmetric aircraft, etc.
- Fluid properties: ρ, T, p $\quad \bar{V} = (U_\infty + u_x)\bar{\imath} + u_y\bar{\jmath} + u_z\bar{k}$

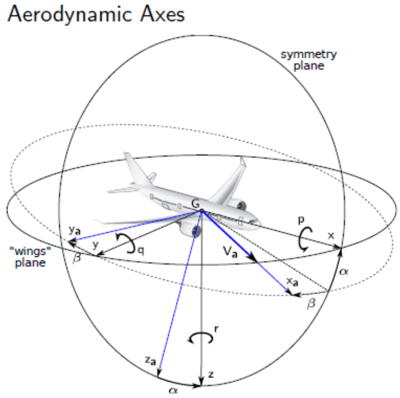
INTRO – GENERAL EQUATIONS

IMPORTANT REFERENCE FRAMES

Body Axes (Aircraft attitude)



Aerodynamic Axes



- ψ: yaw angle or rhumb
- θ: pitch angle
- ϕ: roll angle

- α: angle of attack
- β: sideslip angle

(Mellibovsky, F.P., *Flight Principles*, 2016)

INTRO – GENERAL EQUATIONS

Origin: Gravity Centre.

Earth Axes:

- z_e: Vertical axis. Oriented by local gravitational acceleration vector.
- x_e: Latitude axis. South-North direction.
- y_e: Longitude axis. West-East direction.

Body Axes:

- x: Fuselage (roll) axis. Forward pointing, aligned with the fuselage.
- z: Belly (yaw) axis. In the airplane symmetry plane.
- y: Right wing (pitch) axis. Completing a direct orthonormal basis, aligned with the right wing.

Aerodynamic Axes:

- x_a: Oriented by the aerodynamic velocity vector.
- z_a: In the airplane symmetry plane.
- y_a: Completes a direct orthonormal basis.

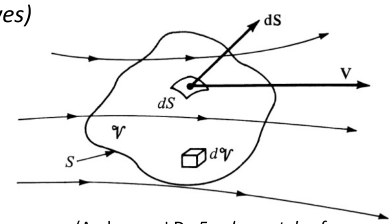
(Mellibovsky, F.P., *Flight Principles*, 2016)

INTRO – GENERAL EQUATIONS

GENERAL EQUATIONS OF THE FLUID MOTION (1)

In aerodynamics, media satisfy the continuum hypothesis:

> ***The fluid is treated as a continuum** by viewing it at a **coarse enough scale** that the airfoil surface cannot distinguish the individual molecular collisions. One can then assign a **local average flow velocity** at a point by averaging over the much faster, violently fluctuating molecular velocities. Similarly, one defines a **locally averaged density, pressure, etc.** These locally averaged quantities then **vary smoothly all over the fluid domain on the macroscopic scale** (except when crossing shock waves)*



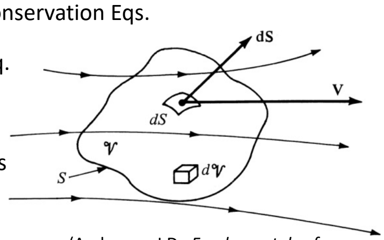
(Anderson, J.D., *Fundamentals of Aerodynamics*, 2001)

INTRO – GENERAL EQUATIONS

GENERAL EQUATIONS OF THE FLUID MOTION (2)

In aerodynamics, continuous media satisfy:

- Law of mass conservation → Continuity Eq.
- Newton's 2nd Principle → Momentum conservation Eqs.
- 1st Law of Thermodynamics → Energy Eq.
- 2nd Law of Thermodynamics
- Constitutive laws → secondary principles that characterize the media:
  - Ex. 1: State Eqs. for fluids
  - Ex. 2: Hooke's Law for linear elastic solids
  - Ex. 3: Navier-Stokes' Law of viscosity, valid for many viscous fluids



(Anderson, J.D., *Fundamentals of Aerodynamics*, 2001)

INTRO – GENERAL EQUATIONS

CONTINUITY EQUATION (1)

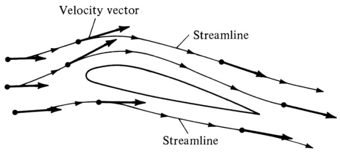
"The net mass flux through the surface of the control vol. is equal to the temporal variation of the mass inside the control vol."

- Integral formulation → $\frac{d}{dt}\int_D \rho dv + \int_S \rho\bar{V}\cdot\bar{n}ds = 0$

Gauss-Ostrogradski (Divergence) Theorem $\int_D \nabla\cdot(\rho\bar{V})dv =$

- Differential formulation → $\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\bar{V}) = \frac{D\rho}{Dt} + \rho\nabla\cdot\bar{V} = 0$

$$\frac{D\rho}{Dt} = \frac{\partial\rho}{\partial t} + \bar{V}\cdot\nabla\rho$$



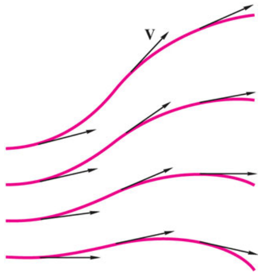
INTRO – GENERAL EQUATIONS

CONTINUITY EQUATION (2)

The substantive or Stokes derivative: $\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \bar{V} \cdot \nabla \rho$

"A derivative taken with respect to a moving coordinate system, also called the Lagrangian derivative, substantive derivative (Tritton 1989), or Stokes derivative (Kaplan 1991, pp. 189-191)."

- **Contribution due to** temporal variation (non-stationary problem) $\frac{\partial \rho}{\partial t}$
- **Contribution due to** variation along the streamline (non-uniform problem) $\bar{V} \cdot \nabla \rho$



Please, note the difference between **streamline** & **particle trajectory**

(White, F.M., *Fluid Mechanics*, 6th ed., Boston, USA: McGraw-Hill, 2003, p. 866)

INTRO – GENERAL EQUATIONS

NEWTON'S 2nd PRINCIPLE (1)

"The temporal variation of momentum of the fluid inside a control vol. is equal to the sum of the external forces acting on the vol."

- Integral formulation

$$\frac{d}{dt}\int_D \rho \bar{V} dv + \int_S \rho \bar{V}(\bar{V} \cdot \bar{n}) ds = -\int_S p\bar{n} ds + \int_S \bar{n} \cdot \tau' ds + \int_D \rho \overline{F_m} dv$$

Gauss-Ostrogradski Theorems

- Differential formulation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{V}) = 0 \longrightarrow \rho \frac{D\bar{V}}{Dt} = -\nabla p + \nabla \cdot \tau' + \rho \overline{F_m}$$

(we simplify with the Continuity Eq.)

INTRO – GENERAL EQUATIONS

NEWTON'S 2nd PRINCIPLE (2)

"The temporal variation of momentum of the fluid inside a control vol. is equal to the sum of the external forces acting on the vol."

- Differential formulation

$$\rho \frac{D\bar{V}}{Dt} = -\nabla p + \nabla \cdot \tau' + \rho \overline{F_m}$$

Note: For a viscous fluid, it is necessary to model the dissipative action of viscosity through the **viscous stress tensor** ⟶ e.g., Navier-Stokes model

$$\tau' = \zeta(\nabla \cdot \bar{V})I + \mu\left[(\nabla\bar{V} + \nabla\bar{V}^T) - (\frac{2}{3})(\nabla \cdot \bar{V})I\right]$$

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INTRO – GENERAL EQUATIONS

ENERGY EQUATION

"In a control vol., there is balance between the temporal variation of total energy & the work produced by external forces plus the heat received from the exterior, per unit time"

- Differential formulation – version 1 (from the 1st Law of Thermodynamics)

$$de = dw + dq \longrightarrow \rho \frac{De}{Dt} = \nabla \cdot (k\nabla T) - p\nabla \cdot \bar{V} + \Phi_v - \nabla \cdot \overline{q_r} + Q_{rq}$$

- Differential formulation – version 2 (we combine the Continuity Eq. & the 2nd Law of Thermodynamics with the previous expression)

$$de = Tds - pdv \longrightarrow \rho T \frac{Ds}{Dt} = \nabla \cdot (k\nabla T) + \Phi_v - \nabla \cdot \overline{q_r} + Q_{rq}$$

$$\Phi_v = \nabla \cdot (\tau' \cdot \vec{v})$$

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INTRO – GENERAL EQUATIONS

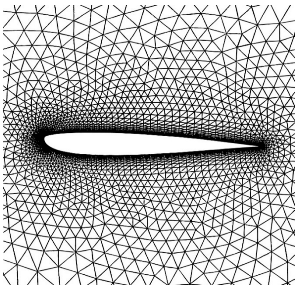
HOW TO SOLVE THE FLUID PROBLEM?

No general analytical (exact) solution satisfies the Navier-Stokes Eqs.

Particular cases allow simplifications: procedure:

- is there any term negligible compared to the others? If so → eliminate it!
- are new Eqs. solvable analytically? only in some simple cases → yes! ☺

  in most cases → still no! ☹

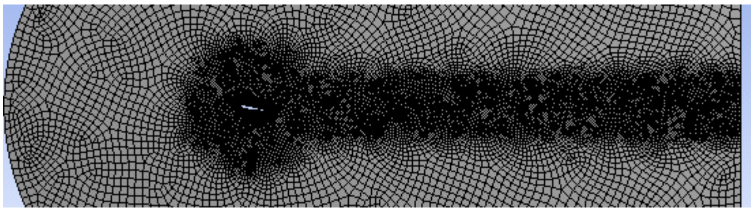


Then, we use numerical methods (the fluid domain is **discretized** into multiple **cells** and we solve the Navier-Stokes Eqs. or simplified versions in, e.g., the **nodes** of these cells):

computational fluid dynamics (CFD)

INTRO – GENERAL EQUATIONS

ORDER OF MAGNITUDE OF TERMS IN EQS.



Newton's 2nd Principle: orders of magnitude:

- pressure term → rel. to inverse of M^2
- viscous term → rel. to inverse of Re
- mass or vol. forces → rel. to inverse of Fr
- non-stationary term → rel. to St

$$M = U_\infty / a$$
$$Re = \rho U_\infty L / \mu$$
$$Fr = {U_\infty}^2 / gL$$
$$St = fL/U_\infty = t_v/t_c$$

$$\rho \frac{D\bar{V}}{Dt} = -\nabla p + \nabla \cdot \tau' + \rho \overline{F_m}$$

INTRO – GENERAL EQUATIONS

SIMPLIFICATIONS IN CONVENTIONAL AERONAUTICS

Newton's 2nd Principle: which terms can we simplify?

- pressure term ⟶ the term we want to compute, as important as the most important term ($M \approx 1$) ⟶ reaction that equilibrates the other terms
- viscous term ⟶ negligible if $Re >> 1$, except in boundary layer (BL) & wake
- mass or vol. forces ⟶ negligible if $Fr >> 1$, i.e., if the flight is due to dynamic effects & not static effects (buoyancy forces/Archimedes' Principle)
  - exceptions ⟶ not negligible for balloons & dirigibles (Zeppelins)

$$\rho \frac{D\bar{V}}{Dt} = -\nabla p + \nabla \cdot \tau' + \rho \bar{F}_m \xrightarrow{Re,\ Fr >> 1;\ M \approx 1} \boxed{\rho \frac{D\bar{V}}{Dt} = -\nabla p}$$

(Euler Eq.)

$$\frac{D\bar{V}}{Dt} = \frac{\partial \bar{V}}{\partial t} + \bar{V} \cdot \nabla \bar{V} = \frac{\partial \bar{V}}{\partial t} + \frac{1}{2}\nabla(\bar{V} \cdot \bar{V}) - \bar{V} \times (\nabla \times \bar{V}) = -\frac{1}{\rho}\nabla p$$

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INTRO – GENERAL EQUATIONS

SIMPLIFICATIONS IN CONVENTIONAL AERONAUTICS

Energy Eq.: orders of magnitude + which terms can we simplify?

- heat flux from radiation ⟶ negligible
- heat flux from chemical reaction ⟶ negligible
  - exception ⟶ not negligible in re-entry of spacecraft ($M > 5$)

$$\rho T \frac{Ds}{Dt} = \nabla \cdot (k \nabla T) + \Phi_v - \nabla \cdot \bar{q}_r + Q_{rq} \longrightarrow \rho T \frac{Ds}{Dt} = \nabla \cdot (k \nabla T) + \Phi_v$$

- heat flux from conduction ⟶ negligible if $Pr \cdot Re >> 1$ $\quad Pr = \mu c_p / k \approx 1$
- Rayleigh dissipation function ⟶ negligible if $Re/M^2 >> 1$

$$\rho T \frac{Ds}{Dt} = \nabla \cdot (k \nabla T) + \Phi_v \longrightarrow \rho T \frac{Ds}{Dt} = 0 \longrightarrow \boxed{\frac{Ds}{Dt} = \frac{\partial s}{\partial t} + \bar{V} \cdot \nabla s = 0}$$

Note: If entropy is constant in the infinite upwind, this constant is the same for all the stream lines, except in boundary layer (BL) & shock waves

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SIMPLIFICATIONS IN CONVENTIONAL AERONAUTICS

Energy Eq. (simplification):

Barotropic relation → unique relation between p & ρ, independent of T:

- incompressible (e.g., liquids) → ρ = constant; independent of p & T
- compressible → only in **isentropic (adiabatic & reversible)** problems:

s = constant → E.g.: perfect gases → $\frac{p}{\rho^{\gamma}} = ct$

State Eq. (constitutive Eq.):

- incompressible (e.g., $M < 0.3$) → $\rho = ct$
- compressible: perfect gases model → $\frac{p}{\rho} = R'T$ $\quad R' = \frac{R}{M}$

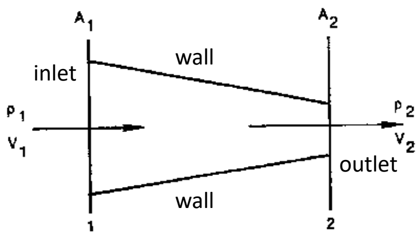
INTRO – GENERAL EQUATIONS

CONTINUITY EQUATION – EXERCISE

"The net mass flux through the surface of the control vol. is equal to the temporal variation of the mass inside the control vol."

- Integral formulation → $\frac{d}{dt}\int_D \rho dv + \int_S \rho\bar{V}\cdot\bar{n}ds = 0$

- Ex.: uniform stationary problem (non-porous walls):
  - case 1: with viscosity
  - case 2: without viscosity
  - case 3: with porous walls

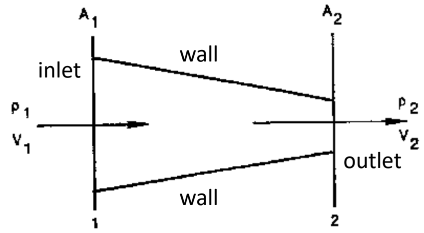


INTRO – GENERAL EQUATIONS

CONTINUITY EQUATION – QUESTION

For an air flow in a duct with *St* << 1 and *M* << 1, if the inlet cross sectional area *A1* is twice as large as that of the outlet *A2*, the inlet airflow velocity:

a) is twice the outlet airflow velocity
b) is equal to the outlet airflow velocity
c) is half the outlet airflow velocity
d) We don't have enough information to choose
e) None of the others



INTRO – GENERAL EQUATIONS

IRROTATIONAL MOTION & VELOCITY POTENTIAL

Circulation & Stokes Theorem: $\Gamma = \oint_C \bar{V} \cdot d\bar{l} = \int_S (\nabla \times \bar{V}) \cdot \bar{n} ds$

Circulation null (global vorticity null) → irrotational motion

Velocity potential: $\Phi = \Phi(x, y, z, t)$ ⟶ $\bar{V} = \nabla\Phi$

Conditions for the velocity potential to exist:

- Bjerknes-Kelvin Theorem: If viscosity & conduction are negligible, mass forces are derived from a potential (e.g., are negligible), & fluid is barotropic → circulation is constant
- If velocity is uniform & stationary (constant) in the infinite (i.e., aircraft flies in steady atmosphere) → circulation is null
- As circulation is constant = 0 → global vorticity must be null for all the fluid field → we have irrotacional motion & velocity potential

INTRO – GENERAL EQUATIONS

EULER-BERNOULLI EQUATION

Euler Eq. particularized for irrotational flow:

$$\frac{\partial \bar{V}}{\partial t} + \frac{1}{2}\nabla(\bar{V}\cdot\bar{V}) - \bar{V}\times(\nabla\times\bar{V}) = -\frac{1}{\rho}\nabla \mathrm{p}$$

$$\nabla\times\bar{V} = 0$$

$$\Phi = \Phi(\mathrm{x},\mathrm{y},\mathrm{z},\mathrm{t})$$

$$\bar{V} = \nabla\Phi$$

(Euler-Bernoulli Eq.)

$$\frac{\partial\Phi}{\partial t} + \frac{1}{2}|\nabla\Phi|^2 + \int\frac{dp}{\rho} = C(t)$$

- incompressible → $\rho = ct$ → $\frac{\partial\Phi}{\partial t} + \frac{1}{2}|\nabla\Phi|^2 + \frac{p}{\rho} = C'(t)$
- perfect gases → $\frac{p}{\rho^\gamma} = ct$ → $\frac{\partial\Phi}{\partial t} + \frac{1}{2}|\nabla\Phi|^2 + \frac{a^2}{\gamma-1} = C''(t)$

INTRO – GENERAL EQUATIONS

BERNOULLI EQUATION

Euler-Bernoulli Eq. particularized for stationary flow:

- incompressible → $\frac{1}{2}\rho_\infty|\nabla\Phi|^2 + p = \frac{1}{2}\rho_\infty {U_\infty}^2 + p_\infty$
- perfect gases → $\frac{1}{2}|\nabla\Phi|^2 + \frac{a^2}{\gamma-1} = \frac{1}{2}{U_\infty}^2 + \frac{{a_\infty}^2}{\gamma-1}$

(Bernoulli Eq.)

$$a = \sqrt{\gamma R'T} = \sqrt{\gamma\frac{p}{\rho}} \qquad R' = \frac{R}{M}$$

INTRO – GENERAL EQUATIONS

PROCEDURE TO FULFIL OUR OBJECTIVE

To compute the aerodynamic forces (*p* distrib.) on the airfoil:

1. compute velocity potential from differential Eq. for velocity potential
2. then compute velocity field from velocity potential
3. then compute pressure field by application of Bernoulli Eq.:

$$\frac{1}{2}\rho_\infty|\nabla\Phi|^2 + p = \frac{1}{2}\rho_\infty {U_\infty}^2 + p_\infty$$

INTRO – GENERAL EQUATIONS

DIFFERENTIAL EQ. FOR VELOCITY POTENTIAL

We obtain the **differential Eq. for the velocity potential** by eliminating ρ & V from the Continuity Eq. + Euler-Bernoulli Eq.

recall that this is only for potential flow ⟶ i.e., Bjerknes-Kelvin Theorem must apply & properties in the infinite are uniform & stationary

Further simplification is possible if we can also assume:

- $St << 1$ ⟶ stationary problem
- low speed: $M < 0.30$ ⟶ compressibility effects neglected: $\rho = ct$

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\bar{V} = 0 \xrightarrow{\rho = ct} \nabla\cdot\bar{V} = 0 \xrightarrow{\bar{V} = \nabla\Phi} \nabla\cdot(\nabla\Phi) = \Delta\Phi = 0$$

(Laplace Eq.) $$\Delta\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0$$

INTRO – GENERAL EQUATIONS

EXERCISE 1.0 – CONTINUITY EQUATION

Escola d'Enginyeria de Telecomunicació i Aeroespacial de Castelldefels
UNIVERSITAT POLITÈCNICA DE CATALUNYA

LESSON 1: INTRO TO AERODYNAMICS - GENERAL EQUATIONS -

THANKS FOR YOUR ATTENTION
ANY QUESTION?

INTRO – GENERAL EQUATIONS

MACH NUMBER (1)

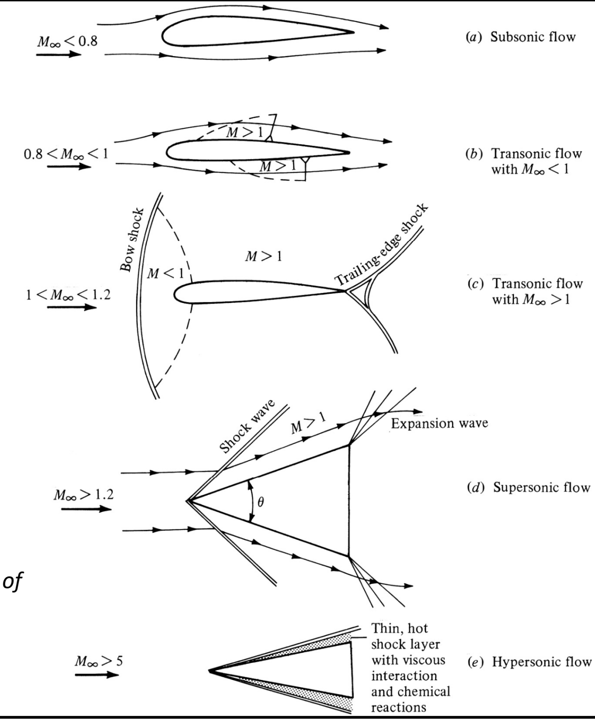
Mach number M: ratio flow velocity-to-speed of sound:

$$M = \frac{U_\infty}{a} \qquad a = \sqrt{\gamma R' T} = \sqrt{\gamma \frac{p}{\rho}} \qquad R' = \frac{R}{M}$$

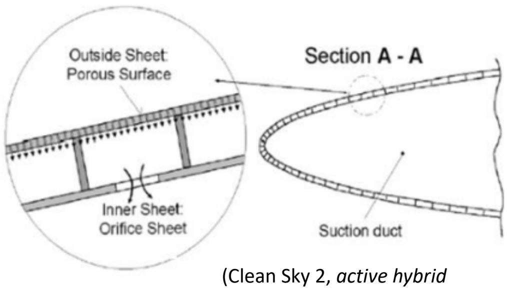
- uncompressible/low subsonic — $M < 0.30$ (or 0.40 or 0.50)
- compressible/high subsonic — $0.30 < M < 0.75$
- transonic — $0.75 < M < 1.20$
- sonic regime — $M = 1.00$
- supersonic — $1.20 < M < 5.00$
- Hypersonic — $M > 5.00$

GENERAL EQS.

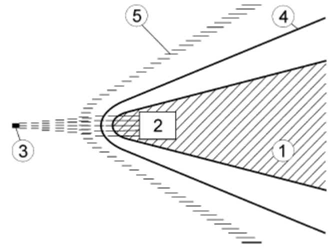
MACH NUMBER (2)



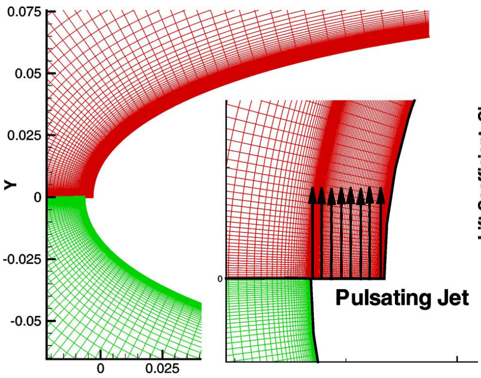
(Anderson, J.D., *Fundamentals of Aerodynamics*, 2001)

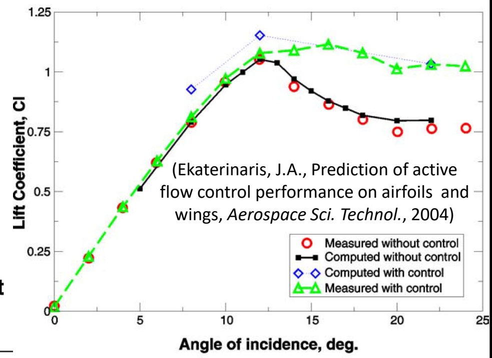


(Clean Sky 2, *active hybrid laminar flow control, HLFC*)



(Fomin, V.M., Flow control using various plasma and aerodynamic approaches, *Aerospace Science and Technology* 8 (2004) 411–421)





(Ekaterinaris, J.A., Prediction of active flow control performance on airfoils and wings, *Aerospace Sci. Technol.*, 2004)

Parameter	Definition	Qualitative ratio of effects	Importance
Reynolds number	$\mathrm{Re} = \frac{\rho U L}{\mu}$	$\frac{\text{Inertia}}{\text{Viscosity}}$	Almost always
Mach number	$\mathrm{Ma} = \frac{U}{a}$	$\frac{\text{Flow speed}}{\text{Sound speed}}$	Compressible flow
Froude number	$\mathrm{Fr} = \frac{U^2}{gL}$	$\frac{\text{Inertia}}{\text{Gravity}}$	Free-surface flow
Weber number	$\mathrm{We} = \frac{\rho U^2 L}{Y}$	$\frac{\text{Inertia}}{\text{Surface tension}}$	Free-surface flow
Rossby number	$\mathrm{Ro} = \frac{U}{\Omega_{\text{earth}} L}$	$\frac{\text{Flow velocity}}{\text{Coriolis effect}}$	Geophysical flows
Cavitation number (Euler number)	$\mathrm{Ca} = \frac{p - p_v}{\rho U^2}$	$\frac{\text{Pressure}}{\text{Inertia}}$	Cavitation
Prandtl number	$\mathrm{Pr} = \frac{\mu c_p}{k}$	$\frac{\text{Dissipation}}{\text{Conduction}}$	Heat convection
Eckert number	$\mathrm{Ec} = \frac{U^2}{c_p T_0}$	$\frac{\text{Kinetic energy}}{\text{Enthalpy}}$	Dissipation
Specific-heat ratio	$k = \frac{c_p}{c_v}$	$\frac{\text{Enthalpy}}{\text{Internal energy}}$	Compressible flow
Strouhal number	$\mathrm{St} = \frac{\omega L}{U}$	$\frac{\text{Oscillation}}{\text{Mean speed}}$	Oscillating flow
Roughness ratio	$\frac{\epsilon}{L}$	$\frac{\text{Wall roughness}}{\text{Body length}}$	Turbulent, rough walls
Grashof number	$\mathrm{Gr} = \frac{\beta \Delta T g L^3 \rho^2}{\mu^2}$	$\frac{\text{Buoyancy}}{\text{Viscosity}}$	Natural convection
Rayleigh number	$\mathrm{Ra} = \frac{\beta \Delta T g L^3 \rho^2 c_p}{\mu k}$	$\frac{\text{Buoyancy}}{\text{Viscosity}}$	Natural convection
Temperature ratio	$\frac{T_w}{T_0}$	$\frac{\text{Wall temperature}}{\text{Stream temperature}}$	Heat transfer
Pressure coefficient	$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2}$	$\frac{\text{Static pressure}}{\text{Dynamic pressure}}$	Aerodynamics, hydrodynamics
Lift coefficient	$C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$	$\frac{\text{Lift force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics
Drag coefficient	$C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics
Friction factor	$f = \frac{h_f}{(V^2/2g)(L/d)}$	$\frac{\text{Friction head loss}}{\text{Velocity head}}$	Pipe flow
Skin friction coefficient	$c_f = \frac{\tau_{\text{wall}}}{\rho V^2/2}$	$\frac{\text{Wall shear stress}}{\text{Dynamic pressure}}$	Boundary layer flow

(White, F.M., *Fluid Mechanics*, 6th ed., Boston, USA: McGraw-Hill, 2003, p. 866)

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