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Universidad Politécnica de Madrid
Programa: Master Universitario en Ingeniería Aeroespacial
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ECUACIONES EN DERIVADAS PARCIALES
Assignment #2

Consider the two-dimensional the two-dimensional wave equation

$$\partial_{tt}u - \Delta u = f(x,y,t) \text{ if } t > 0, \quad u(x,y,0) = \partial_t u(x,y,0) = 0,$$

and heat equation

$$\partial_t u - \Delta u = f(x,y,t) \text{ if } t > 0, \quad u(x,y,0),$$

where the forcing function f is given by

$$f(x,y,t) = \cos 2t \exp[-(2x^2 + 3y^2)/4].$$

1. Solve the wave equation in the whole plane, with $u \to 0$ as $x^2 + y^2 \to \infty$, using the Green function.
2. Solve the wave equation in the domain $\Omega : -1 < x < 1, -1 < y < 1$, with $u = 0$ at $\partial\Omega$, using a spectral representation.
3. For both the unbounded and bounded domain:
   3.1 Elucidate whether the solutions are in phase with the forcing.
   3.2 Compare the CPU time that is required to construct a snapshot of the solution calculated in questions 1 and 2 in the domain Ω in a 100×100 equispaced grid at $t = \pi/2$.
   3.3 Construct the appropriate graphical representations of the solution calculated in questions 1 and 2 to illustrate the solution in the domain Ω as time proceeds.
4. Repeat questions 2, 3, 3.1, and 3.3 for the damped wave equation

   $$\partial_{tt}u + \varepsilon\partial_t u - \Delta u = f(x,y,t) \text{ if } t > 0, \quad u(x,y,0) = \partial_t u(x,y,0) = 0,$$

   with ε =0.01.
5. **(Extra credit)** Repeat questions 1, 2, and 3 (including 3.1, 3.2, and 3.3) for the heat equation.